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SCATTERING THEORIES AND RADAR RETURN

by

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ABSTRACT

The importance and the usefulness of remote sensing has aroused great interest in the investigation of the scattering of waves from rough surfaces. Numerous approaches to the problem are now available for various cases, but none is general and at the same time exact. The underlying principles of the different methods as well as their restrictions are discussed.

The Kirchhoff-Huygens method is used to investigate the scattering from a statistically rough surface in the far zone. Terms that involve the first or second partial derivatives of the surface are all considered and found to be of importance for angles of incidence greater than or equal to 20° in the case of backscattering. The artificial line charge introduced around the edge of the illuminated area to satisfy Maxwell's equations is found to have no effect on the mean return power.

Investigation on the statistical parameters of the surface obtained through fitting experimental curves shows that these quantities are frequency sensitive and are, in general, effective parameters rather than real parameters of the surface. It is shown that the exploring wavelength has a sampling filter effect, i.e., it is sensitive only to a certain range of structure sizes, the experimentally determined range being from less than one to tens of wavelengths. It is also shown that when the incident wavelength is about four times the actual standard deviation of the surface, the statistical parameters obtained through fitting the experimental curves will be the actual ones.

Through the angular variation of the return power it is found that proper representation of the surface-height autocorrelation function will give more information about the surface. Specifically, it is possible to learn the range of structure sizes that are present on a given surface by examining a more detailed surface-height autocorrelation function. The proper form as well as the motivation to it is discussed. A very close fit between theory using such an autocorrelation function and the experimental results (both the moon and the earth data) is obtained over

a range of incident angles from normal incidence to 80° from normal. It is found that most of the contribution near normal incidence is due to that range of the autocorrelation that approximates the slowly varying exponential found alone in several theories, whereas the part of the autocorrelation near the origin that approximates a more rapidly varying exponential governs return at large angles. The autocorrelation differs from the slowly varying exponential only near the origin. Thus, it appears, as is intuitively evident, that large scale features determine the return at near-normal incidence and small-scale features determine that from nearer grazing incidence.

CHAPTER I

INTRODUCTION

The problem of scattering of waves from rough surfaces has been of interest to engineers, physicists, and applied mathematicians for more than sixty years. Although a general and exact solution to this problem is as yet lacking, various special methods that are valid in many cases of practical interest are available. This is especially true when the angle of incidence measured from the vertical is not too large; for then many effects such as polarization, depolarization, shadowing, multiple reflections, etc., may not have come in or become of importance.

In the following chapter we shall survey the scattering theories and state the conditions under which each is valid. The various theories will be discussed under the following four headings: first, the case of surface roughness large compared with the incident wavelength [Feinstein 1954; Daniels 1961; Beckmann 1963; Mitzner 1964; Muhleman 1964, etc.]; secondly, the case of surface roughness small compared with the wavelength [Rice 1951; Miles 1954; Parker 1956; Bass and Bocharov 1958; Mitzner 1964, etc.]; thirdly, the case of surface roughness of assumed shapes [Deriugen 1954; Twersky 1957; Ament 1960, etc.], and lastly exact methods [March 1961]. Roughly speaking, the first case deals with surface irregularities that are large horizontally, and perhaps vertically also, when compared with the incident wavelength. The second case requires the amplitudes of the irregularities to be small compared with the wavelength and slopes small compared with unity. Thus, in units of wavelengths the irregularities have small vertical dimensions but not necessarily small horizontal ones. The third case deals with special surfaces where an exact solution is theoretically possible; no restriction needs to be placed on the size of the irregularities. The exact methods to be discussed are methods of solving a boundary value problem. The results are clearly very complex. However, with high speed computers available, they are not entirely impractical.

In Chapter III, a detailed development of Kirchhoff-Huygens' method of solving the bistatic and monostatic radar return problem is

given. Terms involving the first and the second partial derivatives of the surface, which are usually ignored either partly or completely, [Isakovich 1952; Daniels 1961; Winter 1962; Hagfors 1964] are all evaluated for an exponential surface-height autocorrelation function. The general result is as follows for the average scattered power in all directions,

$$\begin{aligned}
 \bar{P} = & \Re \Re_e \frac{\alpha (1 - \sin^2 \theta \sin^2 \phi)}{\sin \alpha} e^{-k} \left[\frac{2}{\pi \lambda^2 p q} \sin p l \sin q l \right. \\
 & \left. + \sum_{n=1}^{\infty} \frac{k^n}{(n-1)!} \left(\frac{a}{\lambda}\right)^2 R^{-3} \right] \\
 & - \Re \Re_e \left[\frac{8\pi^2 \alpha (\cos \theta + \cos \alpha) (\sin \alpha - \sin \theta \sin \phi) \sin \theta \cos \theta \sin \phi}{\sin \alpha} \right. \\
 & \left. \left(\frac{a \sigma}{\lambda^2}\right)^2 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} R'^{-3} \right] \\
 & + \Re \Re_e \left[\frac{\alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda}\right)^2 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} \left\{ -\frac{n+1}{2R'^3} \right. \right. \\
 & \left. \left. + \frac{1}{4R'^3} \left(\left[\frac{-(n+1)+R'}{a(p-iq)} \right]^2 [(n+1)+2R'] + \left[\frac{a(p-iq)}{-(n+1)+R'} \right] [(n+1)-2R'] \right) \right. \right. \\
 & \left. \left. - \frac{n}{a^2(p+iq)^2} + \frac{1}{2R'} + \frac{1}{4R'} \left(\left[\frac{-(n+1)+R'}{a(p-iq)} \right]^2 + \left[\frac{a(p-iq)}{-(n+1)+R'} \right]^2 \right) \right\} \right] \\
 & + \Re \Re_e \left[\frac{4\pi^2 (\cos \theta + \cos \alpha)^2 \alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda}\right)^4 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} \left\{ \frac{1}{a^2(p+iq)^2} \right. \right.
 \end{aligned}$$

$$-\frac{n+2}{2R''^3} + \frac{1}{4R''^3} \left(\left[\frac{-(n+2) + R''}{a(p-ig)} \right]^2 \left[(n+2) + 2R'' \right] \right. \\ \left. + \left[\frac{a(p-ig)}{-(n+2) + R''} \right]^2 \left[(n+2) - 2R'' \right] \right) \Bigg\}$$

where $\mathbb{E} = 2\eta H_0^2 c \tau S_0 \pi / R_0^2$

$$R = [n^2 + a^2(p^2 + g^2)]^{1/2}$$

$$R' = [(n+1)^2 + a^2(p^2 + g^2)]^{1/2}$$

$$R'' = [(n+2)^2 + a^2(p^2 + g^2)]^{1/2}$$

$$K = k^2 \sigma^2 (\cos\alpha + \cos\theta)^2$$

$$p = k (\sin\alpha - \sin\theta \sin\phi)$$

$$g = k \sin\theta \cos\phi$$

$$k = 2\pi/\lambda$$

l = half the width of the illuminated area

η = the intrinsic impedance of the free space

H_0 = amplitude of the incident H wave

a = correlation distance

c = velocity of light

τ = pulse length

Other symbols are defined in Fig. III-1. It is found that for backscattering the said terms are of importance at large angles of incidence starting at about 20° for the term involving the first partial derivative of the surface and at about 35° for the higher order terms. Mean power expressions for the special case of forward scattering along the specular direction are also given and their variation with the angle of incidence indicates an increase in reflection with incident angle. This behavior checks with experimental results [Taylor 1964].

In the course of the development of the theory of bistatic radar return it is found that the artificial line charge introduced around the edge of the illuminated area has no effect on the mean power scattered.

The comparison between theory and experiment shows that the statistical parameters of the surface obtained from fitting experimental curves are functions of frequency. It is shown in Chapter IV that the incident wavelength is actually sensitive only to a certain range of structure sizes. The experimentally determined range is from less than one to tens of wavelengths [Evans 1962]. Thus, the statistical parameters of the surface obtained this way are effective parameters that characterize only the range of structures seen at the given frequency. Since the standard deviation of a surface is determined mostly by large structures, these effective parameters will coincide with the true parameters of the surface at some frequency that is sensitive to the large structures on the surface. In fact, it is shown that when the exploring wavelength is about four times the standard deviation of the surface, the parameters obtained through fitting the experimental curves will be the true ones. The last statement holds for near-vertical incidence, since at large angles smaller structures will dominate the return [Fung and Moore 1964].

Examining the problem of angular variation of the mean return power shows that a more detailed surface-height autocorrelation function,

$$\begin{aligned}
 P(\xi) = & 1 + K^{-1} \ln \left\{ (c/a) \exp[-K(1 - e^{-|\xi|/L})] \right. \\
 & + \frac{d}{a} \exp[-K(1 - e^{-|\xi|/L'})] + \frac{f}{a} \exp[-K(1 - e^{-|\xi|/L''})] \\
 & \left. + \frac{g}{a} \exp[-K(1 - e^{-|\xi|/L'''})] \right\}
 \end{aligned}$$

where $K = 4 (2\pi/\lambda)^2 \sigma'^2$

σ' = effective standard deviation of the surface heights

λ = wavelength

$a = c + d + f + g$

L, l, l', l'' are the correlation distances of various structures

c, d, f, g are appropriate constants,

is necessary for a surface with continuous distribution of structure sizes. The motivation for its form is discussed in the latter part of Chapter IV. The result shows that with this more detailed autocorrelation function, only the zero order term (the term in the power return expression that does not contain any partial derivative of the surface) needs to be kept within 80° of the vertical. This appears to be a reasonable approximation, since in the region where the zero order term is large, the higher order terms (terms involving the partial derivatives of the surface) are comparatively small. The use of such an autocorrelation function permits a very close fit of both the moon and the earth data over a range of the incident angle from 0° to 80° . It is found that most of the contribution near normal incidence is due to that range of the autocorrelation that approximates the slowly varying exponential found alone in several theories [Daniels 1961; Hayre 1961; Hagfors 1964]; whereas the part of the autocorrelation near the origin that approximates a more rapidly varying exponential governs returns at large angles. This autocorrelation differs from the slowly varying one only in a small region near the origin. Hence, it appears, as is expected intuitively, that large scale features determine the return at near-normal incidence and small-scale features determine that at larger angles.

As will be seen in Chapter III and Chapter IV, in many cases the theory of Chapter III compares favorably with the experimental results. Thus, contribution to mean power return at large angles may be due to

the terms involving the partial derivatives of the surface (see Eq. III-8) rather than the inadequate description of the surface-correlation function by a simple exponential. Further work needs to be done to clarify this point.

A preliminary study on the effects of the size of the illuminated area on radar measurements is also made for the case of near-vertical incidence. It is found that the presence of large undulations comparable in size to the illuminated area will cause a drop in the mean return power. Details are given in section 4.5.

CHAPTER II

LITERATURE SURVEY ON SCATTERING THEORIES

2.1 Introduction

The problem of scattering of waves from a rough surface has been studied continuously since the days of Lord Rayleigh [1895] and has become of special interest during the last twenty years. This is due to its numerous applications in various branches of science such as radar, radio communication, radio astronomy, acoustics, etc. An excellent reference and introduction to the subject is the book by Beckmann and Spizzichino [1963] where both theories and applications are treated. Additional references may be found in survey papers by Lysanov [1958] and Bachynski [1959] and an extensive bibliography is included at the end of this work.

A general and exact solution to the problem is as yet unavailable. This is due to the complications in the boundary conditions which are now functionals of the irregular or random function describing the surface boundary [Rice 1951]. The resulting complexity is always such that approximations must be made whenever an explicit and useful result is desired. The particular type of approximation used depends upon the approach adopted which in turn depends on the type of problem in question. Thus, we can divide the general problem into three different categories where different types of approximations are valid: first, the case when the surface roughness is large compared with the incident wavelength; secondly, when it is small compared with the wavelength, and lastly, when the surface roughness can be replaced by objects of specific shapes. Exact methods have also been developed by some authors, but the result is so complicated that the properties of the solution cannot be deduced except by numerical means. In what follows we shall briefly survey some of the various methods for each of the cases. Others that are modifications of similar methods will be found in the bibliography. Only the basic principle underlying each method will be discussed, but in many cases to enhance understanding, a sketch of the main steps in the development will also be given.

2.2 Case of surface roughness large compared with the wavelength

(i) The Kirchhoff's method

The field scattered from the rough surface is formulated according to Huygens' principle and is given either by the Helmholtz integral (in the scalar case) or the Stratton-Chu integral [Stratton 1941] (in the vector case). These integrals express the scattered field in terms of the total field and its normal derivative or their equivalents on the surface [Silver 1949]. The values of these two quantities are not in general known and are in this case determined by the tangent plane approximation, i. e. the field at each point of the surface may be represented as the sum of the incident wave and a wave reflected from the plane tangent to the surface at the given point. The criterion for the validity of this approximation has been found by Brekhovskikh [1952]. It is

$$4\pi \rho \cos \theta \gg \lambda$$

when the point in question is not a point of inflection, where ρ is the smaller of the two principal radii of curvature at the point; θ is the local angle of incidence and λ is the wavelength of the incident radiation. In the case where the point is a point of inflection, the condition to be satisfied is

$$24\pi^2 \cos \theta \gg \frac{d}{dx} \left(\frac{1}{\rho} \right) \lambda^2$$

where x is the coordinate measured along the mean level of the rough surface [Brekhovskikh 1952].

The above conditions restrict the method to work for locally flat surface composed of irregularities with small curvatures. Also, the angle of incidence must not be near grazing. Within the validity of the basic postulate of the Kirchhoff approximation or the tangent plane approximation, this method gives then an exact solution. However, it is interesting to observe that when the conditions stated above are not satisfied, as for example in the case of surfaces consisting of small rectangular corrugations, this method may still give very good results [Beckmann and Spizzichino 1963, p. 66]. Detailed discussion of this method will be found in Chapter III.

(ii) Muhleman's method

Using ray optics, Muhleman [1964] developed a statistical theory for the radar backscatter angular power function. The physical basis of the theory involves combining two random variables which represent height variations and horizontal scattering lengths to form the probability distribution function for surface slopes. The probability density function of slopes is then shown to be directly related to the backscatter function.

The probability that the normal-to-the surface element lies within a solid angle of $\sin\theta d\theta d\phi$ at an angle θ measured from the normal to the mean spherical surface Figure II-1 is assumed to be of the form (in spherical coordinates)

$$p(\theta, \phi) \sin\theta d\theta d\phi = p(\theta) \sin\theta d\theta \frac{d\phi}{2\pi} \quad (\text{II-1})$$

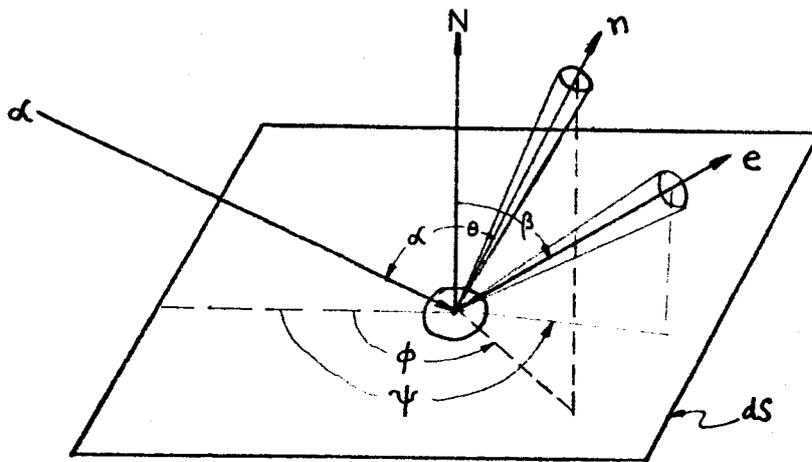


Figure II-1

Geometry defining the incident ray α ; the reflected ray e ; the normal to the scattering element n ; and normal to the mean surface N .

If now a unit flux is incident on an area dS of the mean surface at an angle of incidence, α , from the mean normal, then the intensity δI per unit solid angle scattered into a solid angle $d\Omega$ in the direction (β, ψ) is given by the number of individual scattering elements in dS that are so oriented that the laws of reflection are satisfied. Thus,

$$\delta I d\Omega = \text{Prob} [\underline{n} \text{ in } \sin\theta d\theta d\phi] (\underline{n} \cdot \underline{\alpha}) dS$$

or

$$\delta I \sin\beta d\beta d\psi = \frac{p(\theta)}{2\pi} \sin\theta d\theta d\phi (\underline{n} \cdot \underline{\alpha}) dS \quad (\text{II-2})$$

where (θ, ϕ) are related to (β, ψ) by the laws of reflection; \underline{n} is the normal of a scattering element. Hence, a scattering element will contribute if its normal is in the plane formed by $\underline{\alpha}$ and \underline{e} and midway between $\underline{\alpha}$ and \underline{e} .

By relating $\sin\beta d\beta d\psi$ and $\sin\theta d\theta d\phi$, Equation (II-2) can be reduced for the backscattering case ($\alpha = \beta$, $\psi = 0$) to

$$\delta I = \frac{p(\alpha)}{8\pi} dS \quad (\text{II-3})$$

which states that the probability frequency function of the tilt angles of the scattering elements (slope) is the same as scattering law. This probability function of the tilt angle, α , can be found when some kind of joint probability density is assumed for the horizontal scattering length and height variables. The joint density is then expressed in spherical coordinates in r and α and $p(\alpha)$ is obtained after an integration over r .

For the geometrical laws to apply, this method requires the surface to be covered with plane-scattering elements of unspecified size. However, even so it is not sufficient for the laws of geometrical optics to hold, since these plane scatterers are of finite sizes so that some kind of reradiating pattern rather than a single ray should be considered. Thus, his result predicts an incorrect behavior when compared with the experimental results of Pettit and Nicholson [1931] and Lynn et. al. [1964]. A completely similar idea was employed by Ornstein and Van der Berg [1937] to solve the problem of the scattering of sound from a statistically rough surface.

(iii) The Luneberg-Kline method

This method of analysis bases on expressing the scattered and the transmitted waves at a surface boundary in series expansions in powers of the wavelength. Substituting this series into the wave equation leads to a set of first order linear differential equations in a particular coordinate system. The constants of the solutions to this set of equations are then determined by the boundary conditions.

The series in question is called the Luneberg-Kline series [Jacobson 1962] and it has the form,

$$\underline{\underline{E}}(\underline{r}, \lambda) = \sum_{n=0}^{\infty} \lambda^n \underline{\underline{E}}_n(\underline{r}) e^{ikS(\underline{r})} \quad (\text{II-4})$$

where k and λ are the propagation constant and wavelength respectively. Equation (II-4) was shown by Kline [1951] to be a solution of the vector Helmholtz equation. Thus, the functions $S(\underline{r})$ and $\underline{\underline{E}}(\underline{r})$ are defined by differential equations obtained by substituting (II-4) into the Helmholtz equation and equating like powers of λ . Proceeding in this manner leads to the following set of equations

$$|\nabla S|^2 = 1 \quad (\text{II-5a})$$

$$(\nabla S \cdot \nabla) \underline{\underline{E}}_0 + \frac{1}{2} (\nabla^2 S) \underline{\underline{E}}_0 = 0 \quad (\text{II-5b})$$

$$(\nabla S \cdot \nabla) \underline{\underline{E}}_n + \frac{1}{2} (\nabla^2 S) \underline{\underline{E}}_n = \frac{i}{4\pi} \nabla^2 \underline{\underline{E}} \quad (\text{II-5c})$$

Note that the expression $\underline{\underline{E}}_0 e^{ikS}$ as defined by (II-5a) and (II-5b) constitutes a solution of the zero wavelength limit of Helmholtz's equation. Consequently, it forms a geometrical optics field. Equation (II-5c) shows that the higher order terms of the series give corrections to the geometrical optics field and that they can be obtained by an iterative procedure initiated with $\underline{\underline{E}}_0$. By means of a change of variables [Kline 1951; Jacobson 1962], (II-5) can be solved to give

$$\underline{\underline{E}}_0(S, u, v) = \underline{\underline{C}}_0(u, v) \exp\left[-\frac{1}{2} \int_{s_0}^S \nabla^2 S \, ds\right] \quad (\text{II-6})$$

$$\underline{E}_n(S, u, v) = \frac{i}{4\pi} \left\{ \int_{S_0}^S \nabla^2 \underline{E}_{n-1} \exp\left[\frac{1}{2} \int_{S_0}^S \nabla^2 S \, dS\right] dS + \underline{C}_n(u, v) \right\} \exp\left[-\frac{1}{2} \int_{S_0}^S \nabla^2 S \, dS\right] \quad (\text{II-7})$$

where S_0 is proportional to the phase measured at the boundary and (S, u, v) is the coordinate system defined in part by the distance measured along the geometrical optics rays and in part by (u, v) the point of incidence of a ray on the interface. Note that such a change of variables simplifies (II-5b) and (II-5c) to first order linear differential equations, since $\nabla S \cdot \nabla \rightarrow \partial/\partial S$. The constants $\underline{C}_0(u, v)$ and $\underline{C}_n(u, v)$ are determined from the boundary conditions on $\underline{E}_0(S, u, v)$ and $\underline{E}_n(S, u, v)$ respectively, if one assumes that the integration is to be performed over the rays from the boundary to the point at which the field is to be evaluated. The quantity, $\nabla^2 S$, is given by

$$\nabla^2 S = \frac{1}{h_s h_u h_v} \frac{\partial}{\partial S} \left(\frac{h_u h_v}{h_s} \right) \quad (\text{II-8})$$

where h_s , h_u , and h_v are the square roots of the metric coefficients g_{ss} , g_{uu} , and g_{vv} respectively of the (S, u, v) coordinate system.

The main restriction on this method is the difficulty in computing the metric coefficients from the geometrical structure of the ray system in order to find $\nabla^2 S$. Thus, the problem will be much easier to solve when there is no multiple reflection and shadowing. Clearly, the roughness of the surface in question has to be large compared with the wavelength so that a few terms of the series will suffice. The advantage of this method is that it gives an indication as to how good is the geometrical optics approximation for a given problem. What is more, it supplies all the correction terms.

The extension of this method to the case of random roughness has not been made. It is, however, clear that the restriction on large scale roughness compared with the wavelength cannot be removed.

(iv) The geometrical optics method

The coherency matrix, J , is defined as follows [Born and Wolf 1959]

$$J = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} \quad (\text{II-9})$$

where E_x and E_y are the x- and y- components of the electric vector of an electromagnetic wave traveling in the z - direction, E^* denotes the complex conjugate of E and $\langle \dots \rangle$ indicates time averaging. The trace of the matrix gives the total intensity of the wave and the non-diagonal terms indicate the correlation between the components of the electric vector in the x- and y- directions. Thus, for completely polarized wave, the determinant J is zero; for completely unpolarized wave, the non-diagonal terms are zero and $\langle E_x E_x^* \rangle = \langle E_y E_y^* \rangle$. Other cases then define partially polarized waves.

By calculating the coherency matrix of the reflected wave, Mitzner [1964] solved the problem of a partially polarized wave scattered from a rough plane interface. Both the case where the surface is considered to have a number of specular points and the case where the surface roughness is described statistically were treated.

The problem of reflection from a tilted plane was treated first and leads to the result that the coherency matrix, J^{refl} , of the wave reflected in a given direction is related to the coherency matrix of the incident wave, J^{inc} , by a linear matrix transformation [see Figure II-2]

$$J^{\text{refl}} = P J^{\text{inc}} \tilde{P}^* \quad (\text{II-10})$$

where

$$P = \frac{1}{K} \begin{bmatrix} B_1 B_2 R_{\parallel} + B_3 B_4 R_{\perp} & B_1 B_3 R_{\parallel} - B_2 B_4 R_{\perp} \\ B_2 B_4 R_{\parallel} - B_1 B_3 R_{\perp} & B_3 B_4 R_{\parallel} + B_1 B_2 R_{\perp} \end{bmatrix}$$

\tilde{p}^* = the transpose conjugate of p

$$K = 1 - [\cos \theta \cos \theta' + \cos(\phi' - \phi) \sin \theta \sin \theta']^2$$

$$B_1 = \cos \theta \sin \theta' - \cos(\phi' - \phi) \sin \theta \cos \theta'$$

$$B_2 = \cos(\phi' - \phi) \cos \theta \sin \theta' - \sin \theta \cos \theta'$$

$$B_3 = \sin(\phi' - \phi) \sin \theta'$$

$$B_4 = \sin(\phi' - \phi) \sin \theta$$

R_{\parallel} , R_{\perp} are the Fresnel reflection coefficients for the vertically and horizontally polarized waves respectively

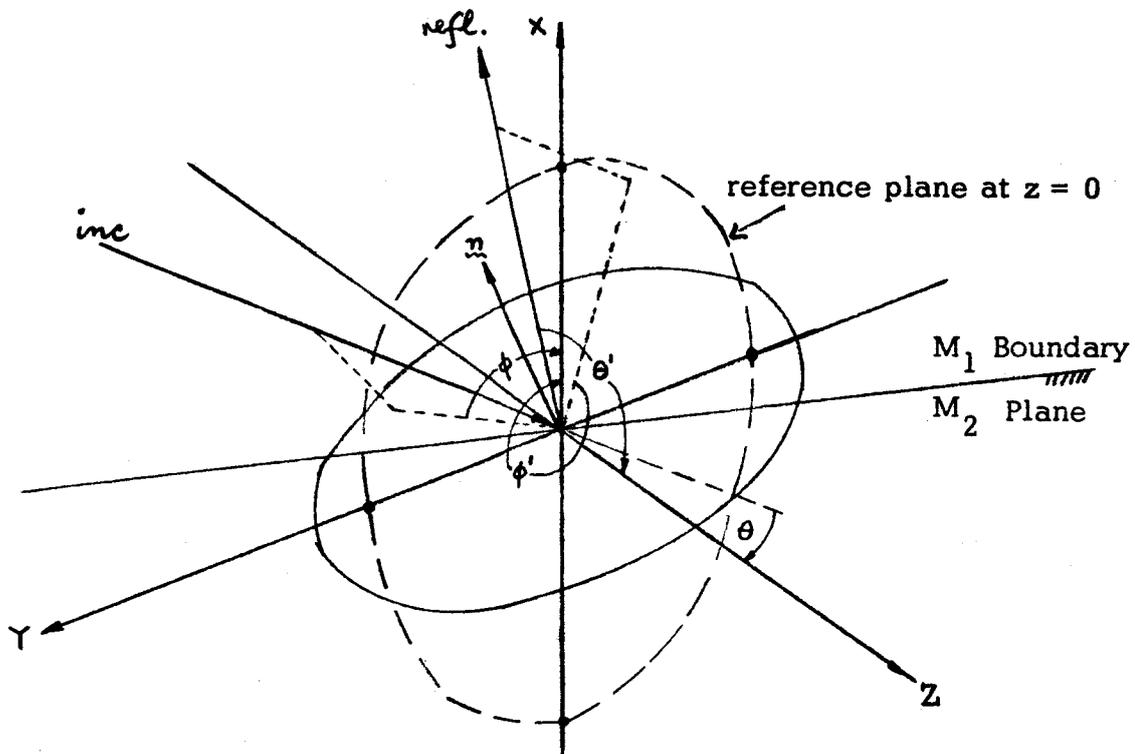


Figure II-2

Geometry of the tilted plane problem

θ, θ' are the angles made by the incident and the reflected rays respectively with the positive Z - axis; ϕ and ϕ' are the corresponding azimuthal angles. In general, the boundary plane is tilted with respect to the reference plane; \underline{n} is a unit vector normal to the boundary plane.

In the far zone and the absence of multiple reflection, shadowing and refraction, (II-10) can be extended to include the case of reflection from a rough plane through a roughness factor η as

$$J^{refl} = \eta P J^{inc} \beta^* \quad (II-11)$$

where
$$\eta = \frac{[1 - \cos\theta \cos\theta' - \cos(\phi' - \phi) \sin\theta \sin\theta']^2}{(\cos\theta - \cos\theta')^4} \cdot \sum_A \frac{1}{|f_{xx} f_{yy} - f_{xy}^2|}$$

The summation is taken over all appropriate specular points in the illuminated area, A ; the f_{xx} , f_{yy} , f_{xy} are the partial derivatives of the surface. For a statistically rough surface, η becomes then a random quantity. The most outstanding feature in this method is that although only laws of geometrical optics are employed, complete information about the state of polarization of the reflected wave is obtained.

2.3 Case of surface roughness small compared with the wavelength

Under this general category we shall describe methods that work for surface irregularities of amplitude small compared with both the wavelength of the incident radiation and the local radii of curvature of the mean surface. Also, the slope of the surface should be much less than unity. These restrictions lead naturally to the method of Rayleigh and the method of perturbation. The latter can also be used to give an approximate solution of an integral equation and thus leads to a different set of restrictions on the method.

(i) The method of small perturbation

The basic concept involved in the treatment of small perturbation is to replace the effect of the surface roughness by an equivalent source distribution on the mean surface. Most of the treatments have been restricted to a perturbed plane surface [Bass and Bocharov 1958; Rice 1951;

Miles 1952; etc.] , but actually it applies to any surface where an appropriate orthogonal curvilinear coordinate system can be used [Mitzner 1964] .

The main steps to be taken in solving an almost plane interface problem are as follows. Let the equation of the interface between two dielectric media be $Z = Z_0(x, y) \equiv Z_0(r_0)$, where r_0 is a point on the unperturbed plane surface, S_0 . Then the perturbed electric field can be written as

$$\underline{E}(r) = \underline{E}^0(r) + \delta \underline{E}(r) + \delta^2 \underline{E}(r) + \dots \quad (\text{II-12})$$

where \underline{E}^0 is the total unperturbed field -- incident plus reflected -- and $\delta^n \underline{E}(r)$ is the perturbation field of order Z_0^n . Let there be no sources in the neighborhood of the interface. Then at a point on the surface, i. e. at $r = i x + j y + k z = r_0 + k Z_0$, Taylor's expansion gives

$$\begin{aligned} \underline{E}(r) = & \underline{E}^0(r_0) + \left[Z_0 \frac{\partial}{\partial Z} \underline{E}^0(r_0) + \delta \underline{E}(r_0) \right] + \left[\frac{1}{2} Z_0^2 \frac{\partial^2}{\partial Z^2} \underline{E}^0(r_0) \right. \\ & \left. + Z_0 \frac{\partial}{\partial Z} \delta \underline{E}(r_0) + \delta^2 \underline{E}(r_0) \right] + \dots \end{aligned} \quad (\text{II-13})$$

Let \underline{n} be the unit local normal vector pointing from medium one to medium two. Then $\underline{E}(r)$ must satisfy the following boundary condition

$$\underline{n} \times (\underline{E}_2 - \underline{E}_1) = \underline{n} \times \Delta \underline{E} = 0$$

$$\text{or} \quad (k - \nabla' Z_0) \times \Delta \underline{E} = 0 \quad (\text{II-14})$$

where $\nabla' \equiv i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y}$.

Substitution of (II-13) into (II-14) leads to

$$\begin{aligned} k \times \left\{ \Delta \underline{E}_0 + \left[Z \Delta \frac{\partial}{\partial Z} \underline{E}^0 + \Delta \delta \underline{E} \right] + \left[\frac{1}{2} Z^2 \frac{\partial^2}{\partial Z^2} \underline{E}^0 + Z \Delta \frac{\partial}{\partial Z} \delta \underline{E} + \Delta \delta^2 \underline{E} \right] \right. \\ \left. + \dots \right\} + \left\{ \Delta \underline{E}_0 + \left[Z \Delta \frac{\partial}{\partial Z} \underline{E}^0 + \Delta \delta \underline{E} \right] \right. \\ \left. + \left[\frac{1}{2} Z^2 \Delta \frac{\partial^2}{\partial Z^2} \underline{E}^0 + Z \Delta \frac{\partial}{\partial Z} \delta \underline{E} + \Delta \delta^2 \underline{E} \right] + \dots \right\} \times \nabla' Z_0 = 0 \end{aligned}$$

$$(\text{II-15})$$

Equating terms of the same order gives

$$\underline{k} \times \Delta \delta \underline{E} = - \underline{k} \times \left[Z \Delta \frac{\partial \underline{E}^0}{\partial Z} + (\Delta E_z) \nabla' z_0 \right] \quad (\text{II-16})$$

$$\underline{k} \times \Delta \delta^2 \underline{E} = - \underline{k} \times \left[Z \Delta \frac{\partial}{\partial Z} \delta \underline{E} + \frac{1}{2} Z^2 \Delta \frac{\partial^2}{\partial Z^2} \underline{E}^0 + (\Delta \delta E_z) \nabla' z_0 \right] \quad (\text{II-17})$$

where the fact that $\Delta \underline{E}_0 \times \nabla' z_0 = \underline{k} \times [\Delta E_z \nabla' z_0]$ and that $\Delta \frac{\partial}{\partial Z} E_z^0 = 0$ on the plane surface is used.

Equations (II-16) and (II-17) give the equivalent magnetic surface currents on the plane surface up to the second order in perturbation. Higher order perturbations can, of course, be determined in the same way. The field everywhere can now be found by using either Kirchhoff's formula [Bass and Bocharov 1958] or dyadic Green's function [Mitzner 1964]. Except that the method is restricted to slightly rough surface, it gives exact solution to the problem and it works also for statistically rough surfaces [Mitzner 1964].

(ii) Rayleigh-Rice method

Rice [1951] gave a direct generalization of the Rayleigh method for solving the scattering problem to the case of a vector wave and a random surface. He treated the problem of a plane wave incident from the dielectric side on an interface between a dielectric and an arbitrary medium. The main idea involved in solving the problem is to assume a representation in series of plane waves for each component of the scattered field with random coefficients. These coefficients are then determined approximately through boundary conditions on the interface and the divergence relation in space. In what follows we outline only the case of scattering from a perfectly conducting surface.

Let the equation of the perfectly conducting rough surface be given by

$$Z = f(x, y) = \sum_{m, n} P(m, n) \exp\{-i a (m x + n y)\} \quad (\text{II-18})$$

$$a = 2\pi / L$$

where the double summation extends from $-\infty$ to ∞ for both m and n , and L is assumed to be very large. The coefficients $P(m, n)$ are taken to be independent random variables subject only to the condition

$$P(-m, -n) = P^*(m, n) \quad (\text{II-19})$$

where the asterick denotes the complex conjugate. This condition is imposed to make $f(x, y)$ real. The coefficients $P(m, n)$ are further assumed to be distributed normally about zero and the four independent random variables formed by the real and imaginary parts of $P(m, n)$ and $P(m, -n)$ all have the same variance. Thus, the following results hold

$$\langle P(m, n) \rangle = 0$$

$$\langle P(m, n) P(u, v) \rangle = 0, \quad (u, v) \neq (-m, -n) \quad (\text{II-20})$$

$$\langle P(m, n) P^*(m, n) \rangle = \pi^2 W(p, q) / L^2$$

$$p = am, \quad q = an$$

Here $\langle \dots \rangle$ denotes that m and n are held fixed and the average taken over the universe of the real and imaginary parts of the $P(m, n)$'s. The reason why the variance is chosen in this way is that, as seen by considering $\langle f^2(x, y) \rangle$ with $L \rightarrow \infty$ and changing the sums into integrals, $W(p, q) dp dq$ represents the contribution to $\langle f^2(x, y) \rangle$ of those components in (II-18) lying between p and $p+dp$ radians/meter in the x -direction and between q and $q+dq$ radian/meter in the y -direction.

With such a model for the surface, the total field in the space $Z > f(x, y)$ corresponding to a horizontally polarized incident wave is written [see Figure II-3]

$$E_x = \sum A_{mn} E(m, n, z) \quad (\text{II-21})$$

$$E_y = 2i \sin \beta \gamma z \exp[-i a \nu x] + \sum B_{mn} E(m, n, z)$$

$$E_z = \sum C_{mn} E(m, n, z)$$

where

$$E(m, n, z) = \exp[-i a (m x + n y) - i b(m, n) z]$$

$$b(m, n) = \begin{cases} [\beta^2 - a^2 m^2 - a^2 n^2]^{1/2}, & m^2 + n^2 < \beta^2 / a^2 \\ -i [a^2 m^2 + a^2 n^2 - \beta^2]^{1/2}, & m^2 + n^2 > \beta^2 / a^2 \end{cases}$$

$$\beta = 2\pi / \lambda$$

λ = incident wavelength

ν is an integer so that the angle of incidence θ between the incoming ray and the z -axis is restricted to certain discrete values given by

$$a \nu = 2\pi \nu / L \equiv \beta \sin \theta \equiv \beta \alpha$$

$$\gamma = \cos \theta, \quad \gamma \geq 0$$

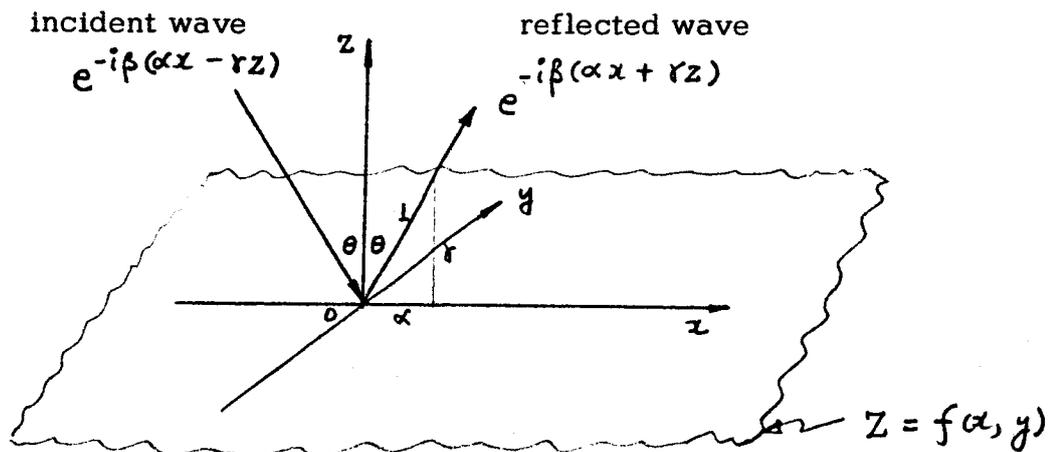


Figure II-3

Geometry of the scattering problem

The coefficients A_{mn} , B_{mn} , C_{mn} can now be determined by the relation, $\nabla \cdot \underline{E} = 0$, which gives

$$a_m A_{mn} + a_n B_{mn} + b(m, n) = 0 \quad (\text{II-22})$$

together with the condition that the tangential component of \underline{E} must vanish on the perfectly conducting surface, i. e.

$$\begin{aligned} E_x - N_x (E_x N_x + E_y N_y + E_z N_z) &= 0 \\ E_y - N_y (E_x N_x + E_y N_y + E_z N_z) &= 0 \end{aligned} \quad (\text{II-23})$$

where N is the unit vector normal to the surface. Now the order of magnitudes of the components of N is

$$N_x = -f_x + O(f^3), \quad N_y = -f_y + O(f^3), \quad N_z = 1 + O(f^2) \quad (\text{II-24})$$

By neglecting terms of order $O(f^3)$ (II-23) becomes

$$\begin{aligned} E_x - N_x E_z &= 0 \\ E_y - N_y E_z &= 0 \end{aligned} \quad (\text{II-25})$$

Approximate the coefficients A_{mn} , B_{mn} , C_{mn} and $E(m, n, z)$ as a sum only of their first and second order terms. Thus, (II-25) becomes

$$\begin{aligned} \sum [A_{mn}^{(1)} + A_{mn}^{(2)} + f_x C_{mn}^{(1)}] [1 - i b(m, n) f] E(m, n, 0) &= 0 \\ 2i \exp[-ia \nu x] \beta \gamma f + \sum [B_{mn}^{(1)} + B_{mn}^{(2)} + f_y C_{mn}^{(1)}] [1 - i b(m, n) f] E(m, n, 0) &= 0 \end{aligned} \quad (\text{II-26})$$

Equating the first and the second order terms to zero leads to

$$\begin{aligned} \sum A_{mn}^{(1)} E(m, n, 0) &= 0 \\ 2i \exp[-ia \nu x] \beta \gamma f + \sum B_{mn}^{(1)} E(m, n, 0) &= 0 \\ \sum [A_{mn}^{(2)} + f_x C_{mn}^{(1)} - i b(m, n) f A_{mn}^{(1)}] E(m, n, 0) &= 0 \\ \sum [B_{mn}^{(2)} + f_y C_{mn}^{(1)} - i b(m, n) f B_{mn}^{(1)}] E(m, n, 0) &= 0 \end{aligned} \quad (\text{II-27})$$

After equating the coefficients of $E(m, n, 0)$ to zero and using the identity,

$$\sum_{m,n} \begin{pmatrix} f \\ f_x \\ f_z \end{pmatrix} J_{mn} E(m, n, 0) = \sum_{m,n,k,l} \begin{pmatrix} 1 \\ -ia(m-k) \\ -ia(n-l) \end{pmatrix} J_{kl} P(m-k, n-l) E(m, n, 0)$$

the coefficients $A_{mn}^{(1)}$, $A_{mn}^{(2)}$, $B_{mn}^{(1)}$ and $B_{mn}^{(2)}$ are found to be

$$A_{mn}^{(1)} = 0$$

$$A_{mn}^{(2)} = \sum_{k,l} ia(m-k) C_{kl}^{(1)} P(m-k, n-l)$$

$$B_{mn}^{(1)} = -2i\beta\gamma P(m-l, n)$$

$$B_{mn}^{(2)} = \sum_{k,l} [ia(n-l)C_{kl}^{(1)} + 2\beta\gamma b(k,l) P(k-l, l)] P(m-k, n-l) \quad (\text{II-28})$$

where $C_{kl}^{(1)}$ can be expressed in terms of $B_{mn}^{(1)}$ through (II-22) giving

$$C_{nl}^{(1)} = -an B_{mn}^{(1)} / b(m, n)$$

Hence, the field components are obtained by substituting (II-28) into (II-21),

$$E_x = -2\beta\gamma \sum_{mn} E(m, n, z) \sum_{kl} a^2(m-k)l Q(m, n, k, l)$$

$$E_y = 2i \exp[-i\beta\alpha z] \sin\beta\gamma z - 2\beta\gamma \sum_{mn} E(m, n, z) [iP(m-l, n) + \sum_{kl} \{a^2(n-l)l - b^2(k, l)\} Q(m, n, k, l)]$$

$$E_z = 2\beta\gamma \sum_{mn} [E(m, n, z) / b(m, n)] [ian P(m-l, n) + \sum_{kl} \{a^3 l(m^2 + n^2 - mk - nl) - an b^2(k, l)\} Q(m, n, k, l)] \quad (\text{II-29})$$

where $Q(m, n, k, l) = P(k-l, l) P(m-k, n-l) / b(k, l)$

The average and the mean square value of the field corresponding to a random non-periodic surface can now be determined using (II-29) and the statistical properties of the surface.

Thus,

$$\langle E_x \rangle = 0$$

$$\begin{aligned} \langle E_y \rangle &= 2i \exp[-i\beta\alpha x] \sin\beta r z \\ &+ 2\beta r E(\nu, 0, z) \sum_{k, l} \left[\frac{a^2 l^2}{b(k, l)} + b(k, l) \right] \frac{\pi^2}{L^2} W(ak - al, al) \\ &\rightarrow \exp[-i\beta(\alpha x - rz)] - \exp[-i\beta(\alpha x + rz)] \cdot \left\{ 1 \right. \\ &\quad \left. - 2\beta \int_{-\infty}^{\infty} dr \int_{-\infty}^{\infty} ds \left[\frac{rs^2}{b} + rb \right] \frac{N(r - \beta\alpha, s)}{4} \right\} \end{aligned}$$

$$\langle E_z \rangle = 0$$

(II-30)

where $a\nu = \beta\alpha$

$$r = ak$$

$$s = al$$

$$b = \begin{cases} [\beta^2 - r^2 - s^2]^{1/2} & , \quad \beta^2 > r^2 + s^2 \\ -i [r^2 + s^2 - \beta^2]^{1/2} & , \quad \beta^2 < r^2 + s^2 \end{cases}$$

In going from the summation to the integration, L was assumed to approach infinity.

$$\langle |E_x|^2 \rangle = 0$$

$$\langle |E_y - 2i \exp[-i\beta\alpha x] \sin\beta r z|^2 \rangle$$

$$= 4\beta^2 \gamma^2 \sum_{m,n,k,l} E^*(m,n,z) E(k,l,z) \langle P^*(m-l,n) P(k-l,l) \rangle$$

$$= 4\beta^2 \gamma^2 \sum_{k,l} \exp[-2\varphi(k,l)] \pi^2 W(ak - al, al) / L^2$$

$$\rightarrow 4\beta^2 \gamma^2 \int_{-\infty}^{\infty} dr \int_{-\infty}^{\infty} ds e^{-2\varphi} W(r - \beta\alpha, s) / 4$$

where $\varphi(k, l) = i b(k, l) - i b^*(k, l)$

$$= \begin{cases} 0, & k^2 + l^2 < \beta^2 / a^2 \\ 2[a^2 k^2 + a^2 l^2 - \beta^2]^{1/2}, & k^2 + l^2 > \beta^2 / a^2 \end{cases}$$

$$\langle |E_x|^2 \rangle \rightarrow 4\beta^2 \gamma^2 \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dq \frac{e^{-2\varphi} q^2 W(p, q)}{4 |b|^2}$$

(II-31)

Other generalization and special cases such as the case of a finite conducting surface, the question of surface waves and a vertically polarized incident wave are also included in Rice's work.

It is interesting to observe that the notion of small perturbation is used in determining the coefficients A_{mn} , B_{mn} , etc. In fact, Lysanov [1955] showed that the results obtained by Rayleigh method and that of small perturbation are identical. However, in some specific problems the perturbation method may prove more convenient from the computation standpoint. The results of Rice can also be shown to be

the same as obtained by the perturbation method [Mitzner 1964]. An approach which is similar but less approximate than Rice has been used by Schouten and Hoop [1957], Kuryenov [1963] and Lapin [1963].

(iii) Integral equation method

The basic features of this method are as follows. A solution of the wave equation for the half space $Z > Z(x)$ is written by means of the Green's formula. The boundary conditions on the rough surface then lead to an integral equation for the field on the surface. It has been shown [Meecham 1956; Lysanov 1955] that for a sufficiently flat absolutely reflecting surface on which the boundary condition, $\phi = 0$, is satisfied for $Z = Z(x)$, the integral equation can be solved approximately. We sketch below the approach of Meecham [1956].

In view of Figure II-4, the field at a point, P, is for a one dimensional surface

$$\Phi(P) = \phi_i(P) + \frac{1}{4i} \int_{Z(x)} \left\{ \phi(z) \frac{\partial}{\partial n_1} H_0^{(1)}(kr_{2p}) - H_0^{(1)}(kr_{2p}) \frac{\partial}{\partial n_2} \phi(z) \right\} dS_z \quad (\text{II-32})$$

where ϕ_i represents the incident wave;

$H_0^{(1)}$, the Green's function, is the zero order Hankel function of the first kind, appropriate to a two-dimensional problem.

$n_{1,2}$ are the unit local normal vectors.

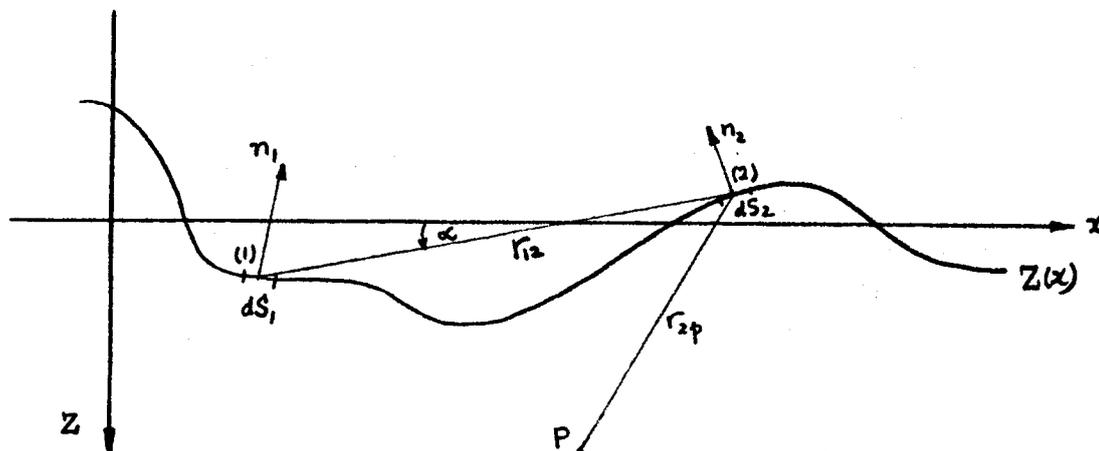


Figure II-4

Diagram used to describe Equation (II-32)

By allowing the point, P, to approach the surface point, (1), and utilizing the boundary condition, $\phi = 0$, (II-32) becomes

$$\begin{aligned}\phi_i(1) &= \frac{1}{4i} \int_{Z(\alpha)} H_0^{(0)}(k r_{12}) \frac{\partial \phi(z)}{\partial n_2} dS_2 \\ &= \frac{1}{4i} \int_{Z(\alpha)} \left[H_0^{(0)}(k|x_1 - x_2|) + K(x_1, x_2) \right] \frac{\partial \phi(z)}{\partial n_2} dS_2\end{aligned}\quad (\text{II-33})$$

where $K(x_1, x_2) = H_0^{(0)}\left(k \frac{|x_1 - x_2|}{\cos \alpha}\right) - H_0^{(0)}(k|x_1 - x_2|)$ and the angle α is defined in Figure II-4.

Under the following two conditions,

$$\left(\frac{dZ^M}{dZ} \right)^2 \ll 1$$

and $kZ^M < 1$, where $k = 2\pi/\lambda$,

$K(x_1, x_2)$ will be small so that the method of perturbation applies.

Z^M , $\frac{dZ^M}{dZ}$ represent the bounds on $Z(x)$ and $\frac{dZ}{dZ}$ respectively.

Let

$$\begin{aligned}\psi(x_2) &= \frac{1}{\cos \chi(x_2)} \frac{\partial \phi(z)}{\partial n_2} \\ F(x_1) &= 4i \phi_i(1)\end{aligned}$$

Note that $K(x_1, x_2)$ is of the first order while $H_0^{(0)}(k|x_1 - x_2|)$ is of the zero order perturbation.

Assume in accordance with small perturbation theory that $\psi(x)$ can be written as a series of terms of different orders of magnitude, i. e.

$$\psi(x) = \psi^{(0)}(x) + \psi^{(1)}(x) + \psi^{(2)}(x) + \dots \quad (\text{II-34})$$

Define $\cos \chi(x_2)$ by the relation,

$$dS_2 = dx_2 / \cos \chi(x_2).$$

Then, the following set of equations hold,

$$\begin{aligned}
\int_{-\infty}^{\infty} H_0^{(1)}(k|x_1-x_2|) \psi^{(0)}(x_2) dx_2 &= F(x_1) \\
\int_{-\infty}^{\infty} H_0^{(1)}(k|x_1-x_2|) \psi^{(1)}(x_2) dx_2 &= - \int_{-\infty}^{\infty} K(x_1, x_2) \psi^{(0)}(x_2) dx_2 \\
\int_{-\infty}^{\infty} H_0^{(1)}(k|x_1-x_2|) \psi^{(2)}(x_2) dx_2 &= - \int_{-\infty}^{\infty} K(x_1, x_2) \psi^{(1)}(x_2) dx_2 \\
&\vdots \\
&\vdots \\
&\vdots
\end{aligned}$$

(II-35)

This set can be solved by a method due to Levi-Civita [1895]. With $\psi(x)$ found, the problem is solved.

It should be pointed out that like the previous two methods, the vertical roughness is required to be small compared with the wavelength, but unlike the other methods, the horizontal scale roughness is restricted only by the condition, $(\frac{dz}{dx})^2 \ll 1$. Hence, it has the advantage over the Rayleigh method as well as the perturbation method in that the error incurred through its use is of second order in the slope of the reflecting surface while for the other two methods it is of the first.

2.4 Case of surface roughness of assumed shapes

In this case the scattering problem is treated by assuming that the surface corrugations possess simple shapes. Then the problem becomes a boundary value problem that can be solved either exactly or approximately. The main advantage in such a treatment is that it facilitates a study of the transition from short wavelength to long wavelength conditions and in some case exact theoretical investigation of polarization problems.

(i) Twersky's method

A method of determining the reflection coefficient, R , and the differential scattering cross section per unit area, σ , of a random distribution of arbitrary bosses on a ground plane was devised by Twersky [1956]. The analysis is based on a Green's function formulation of the problem of a single boss; R and σ then follow from an approximation of the ensemble averaged energy flux which takes account of multiple coherent scattering. The final form of σ and R were found in terms of the scattering amplitude of an isolated boss, their average number in unit area, and the given incident wave. Explicit expressions are obtainable for arbitrary hemispheres and circular semi-cylinders.

For the case of a single boss consider the two dimensional problem of the scattering of a plane wave by a cylinder parallel to the Z - axis (Figure II-5).

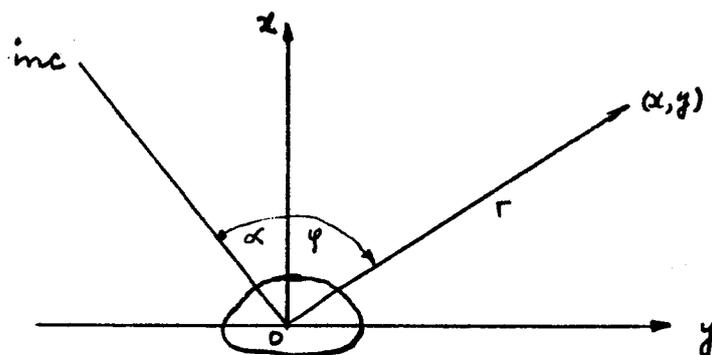


Figure II-5

The solution to the equation

$$(\nabla^2 + k^2) \psi(\underline{r}) = 0 \quad (\text{II-36})$$

$$\nabla^2 = \partial_x^2 + \partial_y^2$$

$$k = 2\pi / \lambda$$

is sought subject to prescribed boundary conditions on the cylinder's surface. As $r \rightarrow \infty$ the scattered wave should vanish and ψ , therefore, reduces to the incident plane wave,

$$\begin{aligned}\psi_0(\underline{r}, \alpha) &= e^{ikx \cos \alpha +iky \sin \alpha} \\ &= e^{ikr \cos(\varphi - \alpha)}\end{aligned}$$

where $r^2 = x^2 + y^2$ and $\varphi = \tan^{-1}(y/x)$. Also, the two dimensional radiation condition requires the difference, $u = \psi - \psi_0$, to be an outgoing cylindrical wave. Thus,

$$\psi(\underline{r}) = \psi_0(\underline{r}) + u(\underline{r}) \quad (\text{II-38})$$

where $u(\underline{r})$ is the scattered wave. Recall that the Green's function for the two dimensional problem is the zero order Hankel function of the first kind, $H_0^{(1)}$ [Morse and Feshbach]. Now, apply Green's theorem to $u(\underline{r})$ and $i H_0^{(1)}(k|\underline{r} - \underline{r}'|)/4$; $\underline{r}(r, \varphi)$ and $\underline{r}'(r', \varphi')$ label a field point and a point on the scatterer's surface respectively.

Integrating over a volume external to the scatterer leads to

$$u(\underline{r}) = \frac{1}{4i} \oint [H_0^{(1)}(k|\underline{r} - \underline{r}'|) \partial_n \psi(\underline{r}') - \psi(\underline{r}') \partial_n H_0^{(1)}(k|\underline{r} - \underline{r}'|)] dA \quad (\text{II-39})$$

For convenience we use Twersky's notation and write (II-39) as

$$u(\underline{r}) \equiv \{H_0^{(1)}(k|\underline{r} - \underline{r}'|), \psi(\underline{r}')\} \quad (\text{II-39})$$

where the integral is over the scatterer's surface, and n is the outward normal. (Note that $\{H_0^{(1)}(k|\underline{r} - \underline{r}'|), \psi_0(\underline{r}')\} = 0$).

In the far zone, the following approximation holds,

$$H_0^{(1)}(k|\underline{r} - \underline{r}'|) \sim (2/i\pi kr)^{1/2} \exp[ikr - ikr' \cos(\varphi' - \varphi)]$$

$$u(\underline{r}) \sim H(kr) g(\varphi, \alpha)$$

$$\text{where } H(kr) = (2/i\pi kr)^{1/2} e^{ikr}$$

$$g(\varphi, \alpha) = \{e^{-ikr' \cos(\varphi' - \varphi)}, \psi(\underline{r}, \alpha)\}$$

Since scalar wave is being discussed here, one can use an image technique due to Rayleigh [1907] -- who showed that the field

scattered by a circular semicylinder protruding from a conducting plane under an incident plane wave equals the field scattered by an entire cylinder in free space illuminated by two incident waves: the originally incident wave and its image with respect to the plane. This technique shows that the scattering amplitudes of a boss on a rigid (+) or free (-) plane $x = 0$ are (For the analogous vector case see Twersky, 1957)

$$f_{\pm}(\varphi, \pi - \alpha) \equiv g(\varphi, \alpha) \pm g(\varphi, \pi - \alpha) = f_{\pm}(\pi - \varphi, \alpha) \quad (\text{II-40})$$

Thus, the total wave functions for the boss problem are given by

$$u_{\pm} \sim H(kr) f_{\pm}(\varphi, \pi - \alpha) \quad (\text{II-41})$$

If there are various cylinders distributed along the plane $x = 0$ with their axes parallel to the Z -axis, then the total field can be assumed to be a plane wave plus a superposition of waves scattered by individual cylinders, i. e.

$$\begin{aligned} \Phi(\underline{r}) &= \Psi_0(\underline{r}) + \sum_s u_s(\underline{r} - \underline{r}'_s) \\ &\equiv \Psi_0(\underline{r}) + U(\underline{r}) \end{aligned} \quad (\text{II-42})$$

where $u_s(\underline{r} - \underline{r}'_s) = \{ H_0(k|\underline{r} - \underline{r}'_s|), \Phi(\underline{r}'_s, \alpha) \}$;
 $\Phi(\underline{r}'_s)$ is the total field at a point \underline{r}'_s on the surface.

Consider as before the far zone case. The asymptotic forms for

$u_1(\underline{r})$ is

$$\begin{aligned} u_1(\underline{r}) &\sim H(kr) G_1(\varphi, \alpha) \\ G_1(\varphi, \alpha) &\equiv \{ e^{-ikr' \cos(\varphi' - \varphi)}, \Phi(\underline{r}'_s, \alpha) \} \end{aligned}$$

where G_1 , the "multiple scattered amplitude of cylinder 1 of the configuration," is a function of the positions of all scatterers because Φ is the total field including effects of other scatterers.

For a single configuration, the total time averaged energy flux per unit area divided by the time-averaged incident flux density is according to Twersky [1957, 1959, 1962]

$$\begin{aligned} \underline{S} &= \text{Re} [\Phi^* \Phi \hat{r}] \\ &= \text{Re} [\Phi^* \nabla \Phi / ik] \end{aligned} \quad (\text{II-43})$$

where $\hat{r} = \underline{r} / r$, a unit vector.

Re denotes the real part.

* denotes the complex conjugate.

The ensemble average of the reflected part of \underline{S} , \underline{S}_r , is then found to be [Twersky 1957]

$$\langle \underline{S}_r \rangle = R(\underline{k}_o, \underline{k}_i) \underline{k}_o + \iint \frac{\sigma(\underline{S}, \underline{k}_i)}{|\underline{r} - \underline{r}_s|} \underline{S} dy_s dz_s \quad (\text{II-44})$$

$$r_s^2 = y_s^2 + z_s^2, \quad S = \frac{\underline{r} - \underline{r}_s}{|\underline{r} - \underline{r}_s|}$$

where \underline{k}_i and \underline{k}_o are the directions of incidence and specular reflection, and where \underline{S} is a unit vector from a point on the distribution \underline{r}_s to the observation point \underline{r} . The function R is the coherently reflected power density, and $\sigma(\underline{S}, \underline{k}_i)$ is the incoherent power scattered into unit solid angle around \underline{S} by unit area of surface. For a uniformly random distribution of identical bosses on a free or rigid base plane, the following expressions for R , σ are obtained on neglecting incoherent multiple scattering,

$$R(\underline{k}_o, \underline{k}_i) = \left| \frac{1+Z}{1-Z} \right|^2, \quad Z = \frac{\pi \rho f(\underline{k}_o, \underline{k}_i)}{k^2 \underline{i} \cdot \underline{k}_o}$$

$$\sigma(\underline{S}, \underline{k}_i) = \frac{\rho}{k^2} \left| \frac{f(\underline{S}, \underline{k}_i)}{1-Z} \right|^2$$

(II-45)

where ρ is the average number of scatterers in unit area and $k = 2\pi/\lambda$;

\hat{i} is the unit vector in x - direction. Note $f(\underline{k}_o, \underline{k}_i) = f(\varphi, \pi - \alpha)$ since either $\underline{k}_o, \underline{k}_i$; or $\varphi, \pi - \alpha$ can be used to denote the directions of incidence and reflection.

Equations (II-44), (II-45) give results in the general form where the exact form of $f(\underline{k}_o, \underline{k}_i)$ is not known. For the particular case of semi-hemispheres or semi-cylinders with large separation distances between them, the specific form for $f(\underline{k}_i, \underline{k}_o)$ can be found [Twersky 1957]. This method then allows us to take into account multiple scattering and permits exact theoretical investigation of polarization problems.

Though not mentioned in the above brief survey of the concept used in the above method, this method allows investigation also of the transmission problem of a random screen. It can be extended to treat distributions of non-identical scatterers [Twersky 1957].

(ii) Deriugen's method

Deriugen [1954] investigated the problem of plane wave scattering from a periodic surface with rectangular grooves. His method of approach is as follows: the region containing the grooves is treated separately from the region above it; solution to the wave equation is then sought in each region and these solutions must match at the imaginary plane boundary between the two regions. This matching at the boundary leads to an infinite system of linear algebraic equations for the amplitudes of the scattered waves. This system of equations is solvable by the method of successive approximations.

In order to obtain a solution to the wave equation in the region containing the grooves, the groove must take on a shape that fits into a separable coordinate system so that the method of separation of variables can be applied. For the case investigated by Deriugen, the general solution in the region containing the grooves will contain plane waves traveling in opposite directions; while the general solution in the region above will contain plane waves either propagated or attenuated in a direction away from the surface, if the incident wave is not counted.

Such an approach permits investigation of the field distribution at the mouth of the grooves [Deriugen 1953] and the phenomenon of surface resonance which occurs when the period of the rough surface is about an integral number of wavelengths.

2.5 Exact solutions

The exact methods to be described here result in very complicated expressions that require high speed computers. These expressions, though useless in providing an indication of the field variations due to the change of a particular parameter, are useful for checking the approximate results obtained by other methods and, of course, are valuable for cases where no approximate methods apply. The essence of such methods is to solve exactly an integral equation that results from the boundary conditions.

(i) Marsh's method

By starting out with a plane wave representation of the scattered wave in integral form, the unknown generalized spectrum of the scattered wave is determined by expanding it in a power series in σ , the rms surface height, and the coefficients involved are then found through a theorem of Wiener in generalized harmonic analysis. This paper by Marsh is quite short and we shall follow through his main development.

Consider a plane wave incident upon an irregular one dimensional surface, $Z(x)$, on which the wave potential vanishes. Then the boundary condition gives

$$g(x, z) e^{i\omega t} + e^{-ik[\alpha x + \gamma Z(x)] + i\omega t} = 0 \quad (\text{II-46})$$

where ω is the angular frequency of the incident wave;

k is the wave number;

α, γ are the direction cosines of incident wave normal with respect to x, z axes respectively;

$g(x, z)$ is the scattered wave except for the time factor.

Now assume a plane wave representation for $g(x, z)$ as follows

$$g(x, z) = \int e^{-i(\lambda \xi - \nu \sigma \xi)} dG(\lambda) \quad (\text{II-47})$$

where $\xi = kx$; $\sigma \xi = kZ(x)$, $\lambda^2 + \nu^2 = 1$, $0 \leq \arg \nu < \pi$;
 $G(\lambda)$ is the generalized spectrum of $g(x, z)$.

The combination of (II-46) and (II-47) gives

$$-e^{-i(\alpha\xi + r\sigma\xi)} = \int e^{-i(\lambda\xi - \nu\sigma\xi)} dG(\lambda) \quad (\text{II-48})$$

for $-\infty < \xi < \infty$.

The problem now is to determine $G(\lambda)$ which is assumed to take the form,

$$G(\lambda) = \sum_0^{\infty} A_m(\lambda) \sigma^m \quad (\text{II-49})$$

Substituting (II-49) in (II-48) leads to

$$\begin{aligned} -e^{-i(\alpha\xi + r\sigma\xi)} &= \int e^{-i(\lambda\xi - \nu\sigma\xi)} \sum_0^{\infty} \sigma^l dA_l(\lambda) \\ -e^{-i\alpha\xi} \sum_0^{\infty} \frac{(-ir\xi)^m}{m!} \sigma^m &= \int e^{-i\lambda\xi} \sum_0^{\infty} \frac{(i\nu\xi)^n}{n!} \sigma^n \sum_0^{\infty} \sigma^l dA_l(\lambda) \\ \sum_{m=0}^{\infty} -e^{-i\alpha\xi} \frac{(-ir\xi)^m}{m!} \sigma^m &= \sum_{m=0}^{\infty} \sum_{n=0}^m \int e^{-i\lambda\xi} \frac{(i\nu\xi)^n}{n!} dA_{m-n}(\lambda) \sigma^m \end{aligned}$$

Equating equal powers of σ leads to

$$e^{-i\alpha\xi} \frac{(-ir\xi)^m}{m!} + \sum_{n=0}^m \int e^{-i\lambda\xi} \frac{(i\nu\xi)^n}{n!} dA_{m-n}(\lambda) = 0 \quad (\text{II-50})$$

Equation (II-50) constitutes an infinite set of simultaneous linear equations for the determination of the $A_m(\lambda)$'s. This set can be solved by using the following relation according to Wiener [1933] which gives the generalized spectrum in wave number space in terms of that wave as

$$G(\lambda) = \lim_{\Lambda \rightarrow \infty} \frac{1}{2\pi} \left[\int_{-\Lambda}^{-1} + \int_{1}^{\Lambda} \right] \frac{f(\xi) e^{i\lambda\xi}}{i\xi} d\xi$$

$$\begin{aligned}
& + \frac{1}{2\pi} \int_{-1}^1 f(\xi) \frac{1 - e^{i\lambda\xi}}{i\xi} d\xi \\
& \equiv W \{ f(\xi) \}
\end{aligned} \tag{II-51}$$

Thus, for $m = 0$, (II-50) gives

$$e^{-i\alpha\xi} = - \int e^{-i\lambda\xi} dA_0(\lambda) \tag{II-52}$$

Note that (II-52) takes the form of (II-47). Hence, by (II-51), $A_0(\lambda)$ is found to be

$$A_0(\lambda) = -W \{ e^{-i\alpha\xi} \} \tag{II-53}$$

Similarly, A_m is found for $m > 0$ to be

$$A_m(\lambda) = -W \left\{ e^{-i\alpha\xi} \frac{(-i\lambda\xi)^m}{m!} - \sum_{n=1}^m W \left\{ e^{-i\lambda\xi} \frac{(i\lambda\xi)^n}{n!} dA_{m-n}(\lambda) \right\} \right\} \tag{II-54}$$

From the above two equations the A_m 's can all be determined. Consequently, $G(\lambda)$ is known.

The same method can be extended to the case of mixed boundary conditions and three dimensional problems. The final answer is in the form of a series of integrals operated upon by the operator defined in (II-51). Thus, it is clear that the expression is quite complicated, but numerical work is possible [Marsh et. al. 1961].

(ii) Another method

The Helmholtz solution to the scalar wave equation gives the total field at a point above the surface, S' . As the observation point approaches the scattering boundary, an inhomogeneous Fredholm integral equation of the first kind is obtained for either the Dirichlet or the Neumann problem. A series solution in terms of a complete set of orthogonal functions is then possible for such an integral equation.

The formal solution for the scattering of a scalar wave by a surface, S' , may be derived from the Helmholtz formula to be [Strutt, J.W. 1945]

$$\phi(P) = \phi_i(P) + \frac{1}{4\pi} \iint_{S'} \left[u(S') \frac{\partial G(P, S')}{\partial n'} - G(P, S') \frac{\partial u(S')}{\partial n'} \right] dS' \quad (\text{II-55})$$

where $G(p, s) = e^{ikr}/R$ is the free space Green's function. $u(S')$ and $\phi(P)$ are the wave potential functions on the surface and at a point P in space respectively. n' is the unit normal on S' ; ϕ_i is the incident wave. On the surface, S' , for Dirichlet problem it is assumed that $u(S') = 0$. This corresponds to a free surface or pressure release surface in the acoustical case or perfectly conducting surface in the electromagnetic case. Then (II-55) becomes

$$\phi_i(S) = \frac{1}{4\pi} \iint_{S'} G(S, S') \frac{\partial u(S')}{\partial n'} dS' \quad (\text{II-56})$$

where S now represents the observation point on the scattering surface (ξ_1, ξ_2, ξ_3) ; and S' the source point on the scattering surface (ξ'_1, ξ'_2, ξ'_3) . Let the mean surface fit into a constant surface of some orthogonal coordinate system. Then dS' can be written as

$$dS' = h_1 h_2 \left[\left(\frac{h_3}{h_1} \frac{\partial S'}{\partial \xi_1} \right)^2 + \left(\frac{h_3}{h_2} \frac{\partial S'}{\partial \xi_2} \right)^2 + 1 \right]^{1/2} d\xi'_1 d\xi'_2 \quad (\text{II-57})$$

where the h_i 's are the scale factors. Hence, (II-56) can be written as

$$\phi_i(S) = \frac{1}{4\pi} \iint_{S'} G(S, S') \psi(S') h_1 h_2 d\xi'_1 d\xi'_2 \quad (\text{II-58})$$

where

$$\psi(S') = \frac{\partial u(S')}{\partial n'} \left[\left(\frac{h_3}{h_1} \frac{\partial S'}{\partial \xi_1} \right)^2 + \left(\frac{h_3}{h_2} \frac{\partial S'}{\partial \xi_2} \right)^2 + 1 \right]^{1/2}$$

Let $g_n(\xi_1, \xi_2)$ be a complete set of normalized functions orthogonal over the region of integration. Then the following expansions are possible

$$\begin{aligned} G(S, S') &= \sum_n b_n(\xi_1, \xi_2) g_n(\xi'_1, \xi'_2) \\ \psi(S') &= \sum_m c_m g_m(\xi'_1, \xi'_2) \end{aligned} \quad (\text{II-59})$$

By substituting (II-59) into (II-58) and integrating over S' , the following expression is obtained

$$\phi_i(s) = \frac{1}{4\pi} \sum_n C_n b_n(\xi_1, \xi_2) \quad (\text{II-60})$$

The problem now is to determine C_n 's. If $b_n(\xi_1, \xi_2)$ is an orthogonal set of functions, the C_n 's can be easily found by quadratures. If not, let u_f be the orthogonal set of functions constructed from the set, b_n , by the Gram-Schmidt procedure [Courant and Hilbert 1937]. Then $\phi_i(s)$ can be expressed in terms of u_f also. Thus,

$$\begin{aligned} \phi_i(s) &= \sum_f a_f u_f = \sum_f a_f \left(\sum_{n=0}^{\infty} \alpha_{fn} b_n \right) \\ &= \frac{1}{4\pi} \sum_n C_n b_n \end{aligned} \quad (\text{II-61})$$

where α_{fn} are coefficients obtained from the Gram-Schmidt procedure and $a_f = \int u_f \phi_i(s) ds$. Hence,

$$C_n = 4\pi \sum_f a_f \alpha_{fn} \quad (\text{II-62})$$

(An explicit expression for the α_{fn} is derived in Morse and Feshbach [1953]).

Thus, the integral equation is solved. From the method of approach, it is clear that the same technique would work for the Neumann problem.

This method is simpler both in principle and in the form of solution than the Marsh's method. However, such a series solution is equally non-informative and numerical means is indispensable.

CHAPTER III

A THEORY OF BISTATIC RADAR RETURN
FROM A STATISTICALLY ROUGH SURFACE3.1 Introduction

A theory is developed here for scatter in any direction of an electromagnetic wave incident upon a statistically rough surface such as the ocean surface or any uniformly rough natural terrain. The Kirchhoff-Huygens scattering theory is used. The surface roughness is described in terms of a Gaussian distribution of heights about the mean surface and an exponential autocorrelation function of height with distance. A unique feature of this development is a coordinate transformation that permits exact evaluation of an integral without the approximation of the autocorrelation function required by techniques of other workers. The terms, which involve the partial derivatives of the surface and have so far been ignored either partly or completely in radar return calculations, are all evaluated. It is shown that these terms give significant contribution for angles of incidence starting at about 20° for the backscattering. It is also shown in the derivation of the Poynting vector that these terms do not result from the artificial discontinuity of surface currents around the edge of the illuminated area.

The results have been specialized to the backscatter case, and compared with lunar as well as earth observations. They appear to fit the data over a wider range of angles than previous theories [Isakovich 1952; Daniels 1961; Hayre 1961; Hughes 1962; Hagfors 1964].

To simplify the results to be derived, we make the following assumptions:

- (1) The surface is perfectly conducting
- (2) There is no overshadowing of one part of the surface by another; there is no multiple reflection.
- (3) The incident electromagnetic plane wave is reflected at every point of the surface as though an infinite plane wave were incident upon the infinite tangent plane at the point.
- (4) The random surface $Z(x, y)$ is continuous in the mean and differentiable over a finite region D .

Except for the tangent plane approximation, other assumptions are not inherent to the Kirchhoff's method [Beckmann and Spizzichino

1963] . However, the resulting ease in obtaining an answer is greatly increased. For a discussion on shadowing and multiple reflection see Beckmann and Spizzichino, 1963; Bass and Fuks, 1963; and Beckmann, 1964.

3.2 The scattered field

Let us assume a time variation of the form, $e^{i\omega t}$, for the incident wave. Then the scattered field at a point P from the surface Z (x,y) becomes (by the scalar-vector analog of Green's theorem [Unz 1958]), except for the time factor,

$$\underline{E}_s = - \frac{i\omega\mu}{4\pi} \int_{A_r} \left[G(\underline{n} \times \underline{H}) - \frac{1}{i\omega\mu} (\underline{n} \cdot \underline{E}) \nabla G \right] dS \quad (\text{III-1})$$

where \underline{E}_s = the scattered electric field

$G = \frac{1}{R} \exp[-i k r]$, the Green's function

$\underline{E}, \underline{H}$ = total electric and magnetic fields

ω = angular frequency of the incident wave

μ = permeability of the space

A_r = the illuminated area

\underline{n}, R are defined in Figure III-1

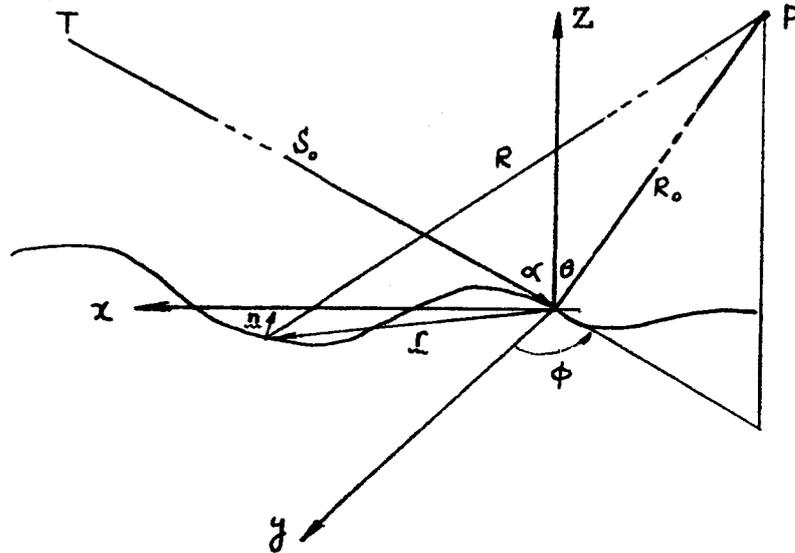


Figure III-1

The incident and scattered waves

Now if there is a boundary line between the illuminated and shadow regions, the current distribution is discontinuous across the boundary. Thus, for the fields to satisfy Maxwell's equations, a line distribution of charge may be introduced along the boundary line so that the source density functions will satisfy the equation of continuity. Assume this is done. Then for the far zone field, (III-1) becomes [see Figure III-1].

$$\underline{E}_s = -\frac{i\omega\mu}{4\pi R_0} e^{-ikR_0} \left\{ \iint_{A_r} \left[\underline{n} \times \underline{H} - \frac{k}{\omega\mu} (\underline{n} \cdot \underline{E}) \underline{R}_0 \right] e^{i\underline{k} \cdot \underline{L}} dS - \frac{k e^{i\underline{k} \cdot \underline{L}}}{\omega^2 \mu \epsilon} \oint \underline{H} \cdot d\underline{l} \right\} \quad \text{(III-2a)}$$

where \underline{R}_0 is a unit vector in R_0 direction $\underline{k} = k \underline{R}_0$ and the line integral is along the boundary of the illuminated area.

Further simplification by means of Stokes theorem and the tangent plane approximation (the details are found in Silver [1949]) leads to

$$\underline{E}_s = -\frac{i\omega\mu}{2\pi R_0} e^{-ikR_0} \int_{A_r} \{ \underline{n} \times \underline{H}_i - [(\underline{n} \times \underline{H}_i) \cdot \underline{R}_0] \underline{R}_0 \} e^{ik \cdot \underline{r}} dS \quad (\text{III-2b})$$

where \underline{H}_i is the incident magnetic field.

As we shall see later, the second term in (III-2b) drops out in the Poynting vector expression, so the additional line integral introduced has no effect on the scattered power.

In the far zone, the scattered electric and magnetic fields are related through

$$\underline{H}_s = \frac{1}{\eta} \underline{R}_0 \times \underline{E}_s \quad (\text{III-3})$$

where η is the intrinsic impedance of free space.

Hence, from (III-1) and (III-3) we have

$$\underline{H}_s = -\frac{i\omega\mu}{2\pi R_0 \eta} e^{ikR_0} \int_{A_r} \underline{R}_0 \times (\underline{n} \times \underline{H}_i) e^{ik \cdot \underline{r}} dS \quad (\text{III-4})$$

3.3 The scattered power

If \underline{E} is polarized in the plane of incidence, then

$$\underline{H}_i = \hat{j} H_0 \exp [ik(x \sin \alpha + z \cos \alpha)] \quad (\text{III-5})$$

The Poynting vector is by definition

$$\begin{aligned} \underline{P} &= \frac{1}{2} \text{Re} (\underline{E} \times \underline{H}^*) \\ &= \frac{1}{2} \text{Re} \left[-\frac{i\omega\mu}{2\pi R_0} e^{-ikR_0} \int_{A_r} \{ \underline{n} \times \underline{H}_i - [(\underline{n} \times \underline{H}_i) \cdot \underline{R}_0] \underline{R}_0 \} e^{ik \cdot \underline{r}} dS \right. \\ &\quad \left. \times \frac{i\omega\mu}{2\pi R_0 \eta} e^{ikR_0} \int_{A_r} \underline{R}_0 \times (\underline{n}' \times \underline{H}_i^*) e^{-ik \cdot \underline{r}'} dS' \right] \end{aligned}$$

In view of (III-5), it becomes

$$\begin{aligned}
 P &= \frac{1}{2} \operatorname{Re} \left[\frac{\omega^2 \mu^2}{(2\pi R_0)^2} \int_{A_r} \left\{ \underline{n} \times \underline{H}_0 - [(\underline{n} \times \underline{H}_0) \cdot \underline{R}_0] \underline{R}_0 \right\} e^{i\mathbf{k} \cdot \underline{r} + i k s} ds \right. \\
 &\quad \left. \times \int_{A_r'} \underline{R}_0 \times (\underline{n}' \times \underline{H}_0) e^{-i\mathbf{k} \cdot \underline{r}' - i k s'} ds' \right] \\
 &\equiv \frac{1}{2} \operatorname{Re} \left[K \iint \left\{ [(\underline{n} \times \underline{H}_0) \cdot (\underline{n}' \times \underline{H}_0)] - (\underline{n}' \times \underline{H}_0) \cdot \underline{R}_0 [(\underline{n} \times \underline{H}_0) \cdot \underline{R}_0] \right\} \underline{R}_0 \right. \\
 &\quad \left. - [(\underline{n} \times \underline{H}_0) \cdot \underline{R}_0 - [(\underline{n} \times \underline{H}_0) \cdot \underline{R}_0]] (\underline{n}' \times \underline{H}_0) \right\} \\
 &\quad \left. \exp[i\mathbf{k} \cdot (\underline{r} - \underline{r}') + i k (s - s')] ds ds' \right] \\
 &= \frac{1}{2} \operatorname{Re} \left[K \iint \left\{ (\underline{n} \times \underline{H}_0) \cdot (\underline{n}' \times \underline{H}_0) - (\underline{n}' \times \underline{H}_0 \cdot \underline{R}_0) (\underline{n} \times \underline{H}_0 \cdot \underline{R}_0) \right\} \underline{R}_0 \right. \\
 &\quad \left. \exp[i\mathbf{k} \cdot (\underline{r} - \underline{r}') + i k (s - s')] ds ds' \right] \quad \text{(III-6)}
 \end{aligned}$$

where $S = x \sin \alpha + z \cos \alpha$

$$K = \frac{\omega^2 \mu^2}{\eta (2\pi R_0)^2}$$

To express in terms of the surface, $Z(x, y)$, we note that

$$ds = [1 + Z_x^2 + Z_y^2]^{1/2} dx dy$$

$$\underline{n} = (-Z_x \hat{x} - Z_y \hat{y} + \hat{z}) / [1 + Z_x^2 + Z_y^2]^{1/2}$$

$$\underline{R}_0 = -\hat{x} \sin \theta \sin \phi + \hat{y} \sin \theta \cos \phi + \hat{z} \cos \theta$$

$$\underline{r} = \hat{x} x + \hat{y} y + \hat{z} z$$

$$\underline{H}_0 = \hat{y} H_0 \quad (\text{III-7})$$

where the quantities with a caret are unit coordinate vectors and Z_x , Z_y are the partial derivatives of $Z(x, y)$.

Thus, using (III-7) we can write (III-6) as

$$\begin{aligned} P &= \frac{1}{2} \text{Re} \left\{ K H_0^2 \iiint \left[(-Z_x \hat{x} - Z_y \hat{y} + \hat{z}) \times \hat{y} \right] \cdot \left[(-Z_x \hat{x} - Z_y \hat{y} + \hat{z}) \times \hat{y} \right] \right. \\ &\quad - \left[\underline{R}_0 \cdot (-Z_x \hat{x} - Z_y \hat{y} + \hat{z}) \times \hat{y} \right] \left[\underline{R}_0 \cdot (-Z_x \hat{x} - Z_y \hat{y} + \hat{z}) \times \hat{y} \right] \\ &\quad \left. \exp [i \underline{k} \cdot (\underline{r} - \underline{r}') + i k (\xi - \xi')] dx dy dx' dy' \right\} \\ &= \frac{1}{2} \text{Re} \left\{ K H_0^2 \iiint \left\{ (Z_x Z_{x'} + 1) - [\sin \theta \sin \phi - Z_x \cos \theta] [\sin \theta \sin \phi \right. \right. \\ &\quad \left. \left. - Z_{x'} \cos \theta] \right\} \exp [i \underline{k} \cdot (\underline{r} - \underline{r}') + i k (\xi - \xi')] dx dx' dy dy' \right\} \\ &= \frac{1}{2} \text{Re} \left\{ K H_0^2 \iiint \left[(1 - \sin^2 \theta \sin^2 \phi) + \cos \theta \sin \theta \sin \phi (Z_x + Z_{x'}) \right. \right. \\ &\quad \left. \left. + \sin^2 \theta Z_x Z_{x'} \right] e^{i k (\xi - \xi')} dx dy dx' dy' \right\} \end{aligned}$$

(III-8)

$$\text{where } \xi - \xi' = (\sin \alpha - \sin \theta \sin \phi) (x-x') + \sin \theta \cos \phi (y-y') \\ + (\cos \theta + \cos \alpha) (z - z')$$

Our interest lies in the mean value of the Poynting vector. To determine it, we shall make use of the Karhunen-Loève theorem in the same manner as was done by Hoffman [1955]. This appears necessary for the evaluation of terms involving the partial derivatives of the surface. The theorem states that a random process defined by the sample function $Z(x, y)$ continuous in the mean on a closed set, D , has on D an orthogonal decomposition

$$Z(x, y) = \sum_{m, n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(x, y) z_{mn}, \quad (x, y) \in D$$

with

$$\int_D \varphi_{mn}(x, y) \varphi_{pq}^*(x, y) dx dy = \delta_{mp} \delta_{nq}$$

$$\overline{z_{mn} z_{pq}^*} = \delta_{mp} \delta_{nq} \quad (\text{III-9})$$

if and only if the λ_{mn} 's are the eigen values and the $\varphi_{mn}(x, y)$ are the orthonormal eigen functions of its correlation function. Then the series converges in the mean on D uniformly. The bar in (III-9) denotes the mathematical expectation. (A proof of the theorem is given in Appendix 1)

By the theorem above, an expression for the autocorrelation function of $Z(x, y)$ in terms of the eigen functions can be found,

$$\rho(x, y; x', y') = \overline{[Z(x, y) Z(x', y')]} \\ = \sum_{mn} \lambda_{mn}^{-1} \varphi_{mn}(x, y) \varphi_{mn}(x', y')$$

(III-10)

Let us now assume $Z(x, y)$ to be a stationary Gaussian random process with zero mean over the set D . It then follows from the theorem that

$$\begin{aligned}\bar{Z}_{mn} &= 0 \\ \sigma_{mn}^2 &= 1\end{aligned}\quad \text{(III-11)}$$

Note that the Z_{mn} 's are Gaussian random variables, since we assume $Z(x, y)$ to be a Gaussian process [Loève 1955]. With the above theorem and (III-10) and (III-11), we can obtain the relations below. (Details are found in Appendix 2)

$$\overline{\exp[ikB(Z - Z')]} = \exp[-k^2\sigma^2B^2(1-r)] \quad \text{(III-12)}$$

$$\overline{Z_x \exp[ikB(Z - Z')]} = ikB\sigma^2 \frac{\partial r}{\partial u} \exp[-k^2\sigma^2B^2(1-r)] \quad \text{(III-13)}$$

$$\overline{Z_{x'} \exp[ikB(Z - Z')]} = ikB\sigma^2 \frac{\partial r}{\partial u} \exp[-k^2\sigma^2B^2(1-r)] \quad \text{(III-14)}$$

$$\begin{aligned}\overline{Z_x Z_{x'} \exp[ikB(Z - Z')]} &= -\left\{\sigma^2 \frac{\partial^2 r}{\partial u^2} + k^2\sigma^4 B^2 \left(\frac{\partial r}{\partial u}\right)^2\right\} \\ &\quad \cdot \exp[-k^2\sigma^2B^2(1-r)]\end{aligned}\quad \text{(III-15)}$$

where r is the autocorrelation coefficient, $u = x' - x$, and B is a function of angles to be defined later.

Applying (III-12) through (III-15) to (III-8), we find the average value of the Poynting vector to be

$$\begin{aligned}\bar{P} &= K_1 \operatorname{Re} \left\{ \iiint \overline{[A' e^{ik(\cos\theta + \cos\alpha)(Z - Z')}]} \right. \\ &\quad + \overline{B'(Z_x + Z_{x'}) e^{ik(\cos\theta + \cos\alpha)(Z - Z')}} \\ &\quad \left. + \overline{C' Z_x Z_{x'} e^{ik(\cos\theta + \cos\alpha)(Z - Z')}} \right\} dx dy dx' dy'\end{aligned}$$

$$= K_1 \operatorname{Re} \left\{ \iiint \left(A' + i 2k B B' \sigma^2 \frac{\partial \Gamma}{\partial u} - c' \sigma^2 \left[\frac{\partial^2 \Gamma}{\partial u^2} + k^2 B^2 \sigma^2 \left(\frac{\partial \Gamma}{\partial u} \right)^2 \right] \right) e^{-k^2 \sigma^2 B^2 (1-r)} dx dy du dv \right. \\ \left. \right. \quad \text{(III-16)}$$

where $K_1 = \frac{1}{2} K H_0^2 = \frac{1}{2 R_0^2 \lambda^2} \eta H_0^2$

$$A' = (1 - \sin^2 \theta \sin^2 \phi) e^{i k v}$$

$$B' = \sin \theta \cos \theta \sin \phi e^{i k v}$$

$$c' = \sin^2 \theta e^{i k v}$$

$$v = -(\sin \alpha - \sin \theta \sin \phi) u - \sin \theta \cos \phi v$$

$$B = \cos \theta + \cos \alpha$$

$$v = y' - y$$

$$u = x' - x$$

$$K = \frac{\omega^2 \mu^2}{\eta (2\pi R_0)^2}$$

3.4 Radar echoes

As an application of (III-16), we consider now the problem of radar returns from a homogeneous statistically rough surface. The assumption of a Gaussian distribution for $Z(x, y)$ about some basic plane is a reasonable one [Hayre 1961; Daniels 1961, 1962]. For the case of a pulse radar we assume in addition the following:

- (a) The variation of the angle of incidence, α , over the domain of integration is negligible.
- (b) The radius of correlation is much smaller than the dimensions of the illuminated area.
- (c) The change in S [see Figure III-1] over the domain of integration is negligible so far as the factor $1/S$ is concerned, but its effect on the phase is taken into account.

(d) The illuminated area is pulse limited.

Since we are going to integrate over the illuminated area, A_r , it is convenient to express the average power in terms of the variables S and γ rather than x and y [see Figure III-2]

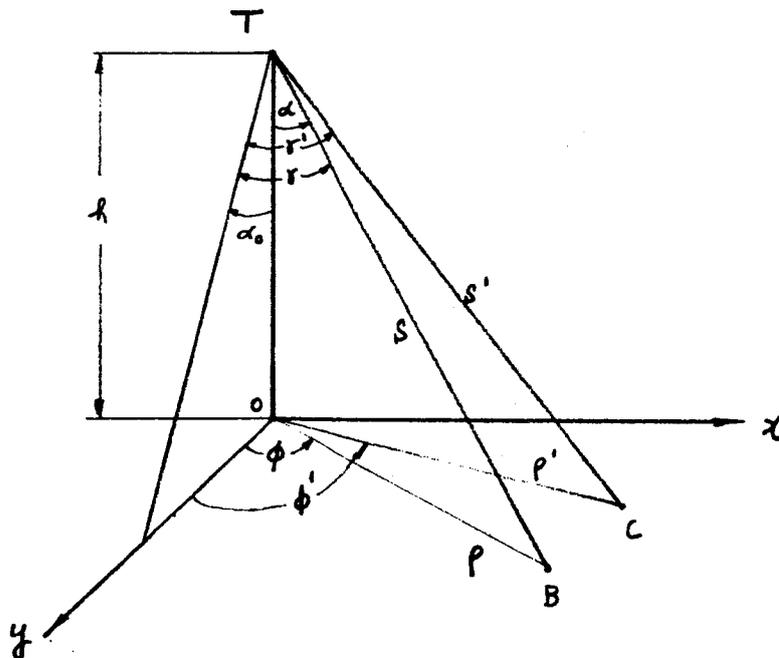


Figure III-2
Geometry of the radar problem

In Figure III-2, we relate S' , γ' to S and γ by

$$S' = S + t \quad , \quad \gamma' = \gamma + \varphi \quad (\text{III-17})$$

We also have from Figure III-2 the following

$$\begin{aligned} x &= P \sin \phi = S \sin \alpha \sin \phi \\ x' &= P' \sin \phi' = S' \sin \alpha' \sin \phi' \end{aligned}$$

$$y = \rho \cos \phi = S \sin \alpha \cos \phi$$

$$y' = \rho' \cos \phi' = S' \sin \alpha \cos \phi'$$

The use of cosine law gives the following relations between ϕ and δ

$$\cos \phi = \frac{\cos \delta - \cos \alpha_0 \cos \alpha}{\sin \alpha_0 \sin \alpha}$$

$$\cos \phi' = \frac{\cos \delta' - \cos \alpha_0 \cos \alpha'}{\sin \alpha_0 \sin \alpha'}$$

It can be shown by choosing $\alpha_0 = 90^\circ$ and ϕ close to 90° that the Jacobian of the system is (see Appendix 3)

$$J \approx S \csc \alpha$$

and

$$u = x' - x \approx \pi \csc \alpha$$

$$v = y' - y \approx -S\psi$$

(III-18)

Observe that in order for ϕ to be a coordinate designating the location of the beam in the ϕ direction, narrow-beam antennas must be used. Our assumption (a) about α , of course, must be satisfied at the same time. It may appear that α will have to be rather large. Actually, what is more important is that the dimension of the illuminated area in the ρ direction should be small compared with ρ . Consequently, it may turn out that assumption (a) holds for $\alpha \geq 1^\circ$ [Moore 1957].

Now in terms of the variables S, r, t, φ , (III-16) becomes

$$\begin{aligned} \bar{P} = & K_1 \operatorname{Re} \left\{ \iiint \left(A_0 + i 2 k B_0 \sigma^2 \frac{\partial r}{\partial (t \csc \alpha)} \right. \right. \\ & \left. \left. - C_0 \sigma^2 \left[\frac{\partial^2 r}{\partial (t \csc \alpha)^2} + k^2 \sigma^2 B^2 \left(\frac{\partial r}{\partial (t \csc \alpha)} \right)^2 \right] \right) \right. \\ & \left. \exp \left[-i p t \csc \alpha + i q S \varphi - k^2 \sigma^2 B^2 (1-r) \right] \right. \\ & \left. S_0^2 \csc^2 \alpha \, dS \, dr \, dt \, d\varphi \right\} \end{aligned}$$

(III-19)

where $A_0 = 1 - \sin^2 \theta \sin^2 \phi$

$$B_0 = (\cos \theta + \cos \alpha) \sin \theta \cos \theta \sin \phi$$

$$C_0 = \sin^2 \theta$$

$$p = k (\sin \alpha - \sin \theta \sin \phi)$$

$$q = k \sin \theta \cos \phi$$

$$S_0 = \text{the mean value of } S$$

The limits on S and r [Davies 1954; Moore 1957] are given by

$$S_0 - \frac{c\tau}{2} \leq S \leq S_0$$

$$-\beta/2 \leq r \leq \beta/2$$

(III-20)

where τ is the pulse length, $\beta/2 = 2 \cos^{-1}(h/S_0)$.

The limits on t and ϕ [Davies 1954; Moore 1957] may be approximated as

$$\begin{aligned} -\infty &\leq t \leq \infty \\ -\infty &\leq \phi \leq \infty \end{aligned} \quad (\text{III-21})$$

To perform the integration it is clear that some form of the autocorrelation coefficient must be assumed. The works of Hayre and Moore [1961], Daniels [1961] and the experimental results of Evans and Pettengill [1963] show that the exponential form gives the best result over the range of the incident angle from 0° to about 25° . Hence, letting $r = \exp[-\sqrt{(S\phi)^2 + (t \csc \alpha)^2} / a]$, where a is the horizontal correlation distance, we obtain from (III-19) the following

$$\begin{aligned} \bar{P} = K_1 R_e \left\{ \iiint \left[A_0 - i 2 k B_0 \sigma^2 \frac{t \csc \alpha}{a \xi} e^{-\frac{\xi}{a}} \right. \right. \\ \left. \left. - C_0 \sigma^2 \left(\left[\frac{t^2 \csc^2 \alpha}{a \xi^3} - \frac{1}{a \xi} + \frac{t^2 \csc^2 \alpha}{a^2 \xi^2} \right] e^{-\frac{\xi}{a}} \right. \right. \right. \\ \left. \left. \left. + k^2 \sigma^2 B^2 \left(\frac{t \csc \alpha}{a \xi} e^{-\frac{\xi}{a}} \right)^2 \right) \right] \right. \\ \left. \cdot \exp \left[-i p t \csc \alpha + i q S \phi - k^2 \sigma^2 B^2 (1 - e^{-\frac{\xi}{a}}) \right] \right. \\ \left. S_0^2 \csc^2 \alpha \quad dS \, d\sigma \, dt \, d\phi \right\} \end{aligned} \quad (\text{III-22})$$

where $\xi^2 = (S\phi)^2 + (t \csc \alpha)^2$.

To evaluate the integrals in (III-22), we make the following change of variables

$$\begin{aligned} S\varphi &= \xi \cos \theta' \\ t \csc \alpha &= \xi \sin \theta' \end{aligned}$$

Then (III-22) becomes

$$\begin{aligned} \bar{P} &= K_1 \operatorname{Re} \left\{ \iiint \left[A_0 - i 2 k B_0 \sigma^2 \frac{\sin \theta'}{a} e^{-\frac{\xi}{a}} \right. \right. \\ &\quad \left. \left. - C_0 \sigma^2 \left(\left[\frac{\sin^2 \theta'}{a^2} - \frac{\cos^2 \theta'}{a \xi} \right] e^{-\frac{\xi}{a}} \right. \right. \right. \\ &\quad \left. \left. \left. + k^2 \sigma^2 B^2 \frac{\sin^2 \theta'}{a^2} e^{-2 \frac{\xi}{a}} \right) \right] \right. \\ &\quad \left. \cdot \exp \left[-i \psi \xi - K \left(1 - e^{-\frac{\xi}{a}} \right) \right] \right. \\ &\quad \left. S_0 \csc \alpha \xi d\xi d\theta' dS dt \right\} \end{aligned}$$

(III-23)

where $\psi = -p \sin \theta' + q \cos \theta'$

$$K = k^2 \sigma^2 B^2$$

Upon expanding $\exp[K \exp(-\xi/a)]$ into a series in $K \exp(-\xi/a)$, we see that the integration with respect to ξ becomes a trivial matter. The integration with respect to θ' can be performed by means of standard contour integration technique by the following change of variables onto a unit circle.

$$\cos \theta' = (z + z^{-1})/2, \quad \sin \theta' = (z - z^{-1})/2i, \quad d\theta' = dz/iz$$

The advantage of evaluating the integral this way is that no approximation needs to be made for the autocorrelation function as in the works of Hayre and Davies; also, the terms (which involve partial derivatives of the random function $Z(x,y)$ in (III-8)) that have been neglected in the works of Winter [1962] and others are evaluable in the same manner. (Details of the evaluation are given in Appendix 4)

The final result of the integration for \bar{P} is as follows (see equations (6), (12), (18) and (24) in Appendix 4)

$$\begin{aligned}
 \bar{P} = & \Re \frac{\alpha A_0}{\sin \alpha} e^{-k} \left[\frac{2}{\pi \lambda^2 \rho \eta} \sin \rho \sin \eta + \sum_{n=1}^{\infty} \frac{k^n}{(n-1)!} \left(\frac{a}{\lambda}\right)^2 Q^{-3} \right] \\
 & - \Re \left\{ \frac{8\pi^2 \alpha B_0 (\sin \alpha - \sin \theta \sin \phi)}{\sin \alpha} \left(\frac{a\sigma}{\lambda^2}\right)^2 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} Q^{-3} \right\} \\
 & + \Re \left\{ \frac{\alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda}\right)^2 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} \left[\frac{1}{a^2 (\rho + i\eta)^2} - \frac{(n+1)}{2} Q^{-3} \right. \right. \\
 & \left. \left. + \frac{1}{4} Q^{-3} \left\{ \left(\frac{Q' - (n+1)}{a(\rho - i\eta)} \right)^2 [n+1 + 2Q'] + \left(\frac{a(\rho - i\eta)}{Q' - (n+1)} \right)^2 [(n+1) - 2Q'] \right\} \right. \right. \\
 & \left. \left. - \frac{n+1}{a^2 (\rho + i\eta)^2} + \frac{1}{2Q'} + \frac{1}{4Q'} \left(\left[\frac{Q' - (n+1)}{a(\rho - i\eta)} \right]^2 + \left[\frac{a(\rho - i\eta)}{Q' - (n+1)} \right]^2 \right) \right] \right\} \\
 & + \Re \left\{ (2\pi)^2 B^2 \frac{\alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda}\right)^4 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} \left[\frac{1}{a^2 (\rho + i\eta)^2} \right. \right. \\
 & \left. \left. - \frac{(n+1)}{2} Q''^{-3} + \frac{1}{4} Q''^{-3} \left\{ \left[\frac{Q'' - (n+1)}{a(\rho - i\eta)} \right]^2 [(n+1) + 2Q''] \right. \right. \right. \\
 & \left. \left. \left. + \left[\frac{a(\rho - i\eta)}{Q'' - (n+1)} \right]^2 [(n+1) - 2Q''] \right\} \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
&= \Re e \frac{\alpha (1 - \sin^2 \theta \sin^2 \phi)}{\sin \alpha} e^{-k} \left[\frac{2}{\pi \lambda^2 p q} \sin p l \sin q l \right. \\
&\quad \left. + \sum_{n=1}^{\infty} \frac{k^n}{(n-1)!} \left(\frac{a}{\lambda}\right)^2 R^{-3} \right] \\
&- \Re e \left[\frac{8\pi^2 \alpha (\cos \theta + \cos \alpha) (\sin \alpha - \sin \theta \sin \phi) \sin \theta \cos \theta \sin \phi}{\sin \alpha} \right. \\
&\quad \left. \cdot \left(\frac{a \sigma}{\lambda^2}\right)^2 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} R'^{-3} \right] \\
&+ \Re e \left[\frac{\alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda}\right)^2 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} \left\{ -\frac{n+1}{2R'^3} \right. \right. \\
&\quad \left. \left. + \frac{1}{4R'^3} \left(\left[\frac{R' - (n+1)}{a(p-iq)} \right]^2 [n+1+2R'] + \left[\frac{a(p-iq)}{R' - (n+1)} \right]^2 [n+1-2R'] \right) \right. \right. \\
&\quad \left. \left. - \frac{n}{a^2(p+iq)^2} + \frac{1}{2R'} + \frac{1}{4R'} \left(\left[\frac{R' - (n+1)}{a(p-iq)} \right]^2 + \left[\frac{a(p-iq)}{R' - (n+1)} \right]^2 \right) \right\} \right] \\
&+ \Re e \left[\frac{4\pi^2 (\cos \theta + \cos \alpha)^2 \alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda}\right)^4 e^{-k} \sum_{n=0}^{\infty} \frac{k^n}{n!} \left\{ \frac{1}{a^2(p+iq)^2} \right. \right. \\
&\quad \left. \left. - \frac{n+2}{2R''^3} + \frac{1}{4R''^3} \left(\left[\frac{R'' - (n+2)}{a(p-iq)} \right]^2 [n+2+2R''] \right) \right. \right. \\
&\quad \left. \left. + \left[\frac{a(p-iq)}{R'' - (n+2)} \right]^2 [(n+2) - 2R''] \right\} \right] \tag{III-24}
\end{aligned}$$

where $\Xi = K_1 \pi \beta c \tau s_0 \lambda^2 / \alpha = 2\eta H_0^2 c \tau s_0 \pi / R^2$

$$R = [n^2 + a^2(p^2 + q^2)]^{1/2}$$

$$R' = [(n+1)^2 + a^2(p^2 + q^2)]^{1/2}$$

$$R'' = [(n+2)^2 + a^2(p^2 + q^2)]^{1/2}$$

$$K = k^2 \sigma^2 (\cos \alpha + \cos \theta)^2$$

$$p = k (\sin \alpha - \sin \theta \sin \phi)$$

$$q = k \sin \theta \cos \phi$$

$$k = 2\pi / \lambda$$

$l =$ half the width of the illuminated area

For the special case of backscattering (i. e. the transmitter and the receiver in the same location with $\phi = -\pi/2$, $\alpha = \theta$), equation (III-24) reduces to

$$\begin{aligned} \bar{P} &= \Xi \frac{\alpha \cos^2 \alpha}{\sin \alpha} e^{-K} \left(\frac{a}{\lambda}\right)^2 \sum_{n=1}^{\infty} \frac{K^n}{(n-1)! R^3} \\ &+ \Xi \alpha \sin \alpha e^{+K} \left(\frac{a}{\lambda}\right)^2 \left[\sum_{n=1}^{\infty} \frac{2K^n}{(n-1)! R^3} + \sum_{n=0}^{\infty} \frac{K^n}{n!} \left\{ -\frac{n+1}{2R^3} \right. \right. \\ &+ \frac{1}{4R^3} \left(\left[\frac{R' - (n+1)}{a(p-iq)} \right]^2 [n+1 + 2R'] \right. \\ &+ \left. \left. \left[\frac{a(p-iq)}{R' - (n+1)} \right]^2 [n+1 - 2R'] \right) \right\} - \frac{n}{a^2(p+iq)^2} \\ &+ \frac{1}{2R'} + \frac{1}{4R'} \left(\left[\frac{R' - (n+1)}{a(p-iq)} \right]^2 + \left[\frac{a(p-iq)}{R' - (n+1)} \right]^2 \right) \left. \right\} \end{aligned}$$

$$\begin{aligned}
& + \int 16\pi^2 \alpha \sin \alpha \cos^2 \alpha e^{-K} \left(\frac{\sigma}{\lambda}\right)^4 \sum_{n=0}^{\infty} \frac{K^n}{n!} \left\{ \frac{1}{a^2(p+iq)^2} \right. \\
& - \frac{n+1}{2R''^3} + \frac{1}{4R''^3} \left(\left[\frac{R'' - (n+1)}{a(p-iq)} \right]^2 [(n+1) + 2R''] \right. \\
& \left. \left. + \left[\frac{a(p-iq)}{R'' - (n+1)} \right]^2 [n+1 - 2R''] \right) \right\} \quad \text{(III-25)}
\end{aligned}$$

$$\text{where now } R = [n^2 + a^2 p^2]^{1/2}$$

$$R' = [(n+1)^2 + a^2 p^2]^{1/2}$$

$$R'' = [(n+2)^2 + a^2 p^2]^{1/2}$$

$$K = 4 k^2 \sigma^2 \cos^2 \alpha$$

$$p = 2k \sin \alpha$$

It is seen that the first term in (III-25) takes the same form as obtained by Hayre [1961] except for a constant that appears in the denominator. We believe that this is due to the fact that Hayre made an approximation to the autocorrelation coefficient before he performed the integration.

For the case of the forward scattering along the direction of specular reflection (III-24) also applies. However, since both p and q are zero in this case, limits must be taken. Thus, with $\alpha = \theta$, $\phi = \pi/2$ and letting $p, q \rightarrow 0$, we obtain the following (see Appendix 5)

$$\begin{aligned}
\bar{P} &= \int \frac{\alpha \cos^2 \alpha}{\sin \alpha} e^{-K} \left[\frac{2l^2}{\pi \lambda^2} + \left(\frac{a}{\lambda}\right)^2 \sum_{n=1}^{\infty} \frac{K^n}{n^2 n!} \right] \\
&+ \int \alpha \sin \alpha \left(\frac{\sigma}{\lambda}\right)^2 e^{-K} \sum_{n=0}^{\infty} \frac{K^n}{2n!} \frac{n}{(n+1)!} \\
&- \int 16\pi^2 \alpha \sin \alpha \cos^2 \alpha \left(\frac{\sigma}{\lambda}\right)^4 e^{-K} \sum_{n=0}^{\infty} \frac{K^n}{2n!} \frac{1}{(n+2)^2} \quad \text{(III-26)}
\end{aligned}$$

The equations obtained above for the mean power are not only complicated, but they also take on the undesirable form of an infinite series. This series can be eliminated if K is sufficiently large. For then the autocorrelation coefficient can be approximated by

$$\exp[-\xi/a] \approx 1 - \xi/a$$

Integration of (III-23) then leads to the following expression for the averaged power (see Appendix 6)

$$\begin{aligned} \bar{P} = & \int \operatorname{Re} \left[\frac{\alpha (1 - \sin^2 \theta \sin^2 \phi)}{\sin \alpha} \left(\frac{a}{\lambda} \right)^2 \frac{K}{D^3} \right. \\ & - \int \operatorname{Re} \left[\frac{8\pi^2 \alpha (\cos \theta + \cos \alpha) (\sin \alpha - \sin \theta \sin \phi) \sin \theta \cos \theta \sin \phi}{\sin \alpha} \right. \\ & \cdot \left. \left(\frac{a\sigma}{\lambda^2} \right)^2 \frac{1}{D^3} \right. \\ & + \int \operatorname{Re} \left[\frac{\alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda} \right)^2 \left\{ -\frac{K'}{2D'^3} + \frac{1}{4D'^3} \left(\left[\frac{-K' + D'}{a(p-iq)} \right]^2 [K' + 2D'] \right. \right. \right. \\ & + \left. \left. \left[\frac{a(p-iq)}{-K' + D'} \right]^2 [K' - 2D'] \right) + \frac{1}{2D'} \right. \\ & \left. \left. + \frac{1}{4D'} \left(\left[\frac{-K' + D'}{a(p-iq)} \right]^2 + \left[\frac{a(p-iq)}{-K' + D'} \right]^2 \right) \right\} \right] \\ & + \int \operatorname{Re} \left[\frac{K\alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda} \right)^2 \left\{ -\frac{K''}{2D''^3} + \frac{1}{4D''^3} \left(\left[\frac{-K'' + D''}{a(p-iq)} \right]^2 [K'' + 2D''] \right. \right. \right. \\ & \left. \left. + \left[\frac{a(p-iq)}{-K'' + D''} \right]^2 [K'' - 2D''] \right) \right\} \right] \end{aligned}$$

where $\bar{K} = 2\eta H_0^2 c \tau s_0 \pi / R_0^2$

$$D = [K^2 + a^2(p^2 + q^2)]^{1/2}$$

$$D' = [K'^2 + a^2(p^2 + q^2)]^{1/2}$$

$$D'' = [K''^2 + a^2(p^2 + q^2)]^{1/2}$$

$$K' = K + 1$$

$$K'' = K + 2$$

$$K = k^2 \sigma^2 (\cos \alpha + \cos \theta)^2$$

$$p = k (\sin \alpha - \sin \theta \sin \phi)$$

$$q = k \sin \theta \cos \phi$$

The special case of backscattering now takes the form

$$\begin{aligned} \bar{P} = & \bar{K} 16\pi^2 \left[\frac{\alpha \cos^4 \alpha}{\sin \alpha} \left(\frac{a\sigma}{\lambda^2} \right)^2 \frac{1}{D^3} + 2\alpha \sin \alpha \cos^2 \alpha \left(\frac{a\sigma}{\lambda^2} \right)^2 \frac{1}{D^3} \right] \\ & + \bar{K} \alpha \sin \alpha \left(\frac{\sigma}{\lambda} \right)^2 \left\{ -\frac{K'}{2D'^3} + \frac{1}{4D'^3} \left(\left[\frac{D' - K'}{ap} \right]^2 [K' + 2D'] \right. \right. \\ & \left. \left. + \left[\frac{ap}{D' - K'} \right]^2 [K' - 2D'] \right) + \frac{1}{2D'} + \frac{1}{4D'} \left(\left[\frac{D' - K'}{ap} \right]^2 + \left[\frac{ap}{D' - K'} \right]^2 \right) \right\} \\ & + \bar{K} 16\pi^2 \alpha \sin \alpha \cos^2 \alpha \left(\frac{\sigma}{\lambda} \right)^4 \left\{ -\frac{K''}{2D''^3} + \frac{1}{4D''^3} \left(\left[\frac{D'' - K''}{ap} \right]^2 [K'' + 2D'] \right. \right. \\ & \left. \left. + \left[\frac{ap}{D'' - K''} \right]^2 [K'' - 2D'] \right) \right\} \end{aligned}$$

(III-28)

$$\text{where now, } D = [K^2 + a^2 p^2]^{1/2}$$

$$D' = [K'^2 + a^2 p^2]^{1/2}$$

$$D'' = [K''^2 + a^2 p^2]^{1/2}$$

$$K = 4k^2 \sigma^2 \cos^2 \alpha$$

$$K' = K + 1$$

$$K'' = K + 2$$

$$p = 2k \sin \alpha$$

For the case of forward scattering along the specular direction, (III-27) reduces to

$$\begin{aligned} \bar{P} &= K \frac{\alpha}{(4\pi)^4 \sin \alpha \cos^2 \alpha} \left(\frac{a\lambda}{\sigma^2} \right)^2 \\ &+ K \frac{1}{2} \alpha \sin \alpha \left(\frac{\sigma}{\lambda} \right)^2 \left\{ \frac{1}{K'} - \frac{1}{K'^2} - \frac{K}{K''^2} \right\} \\ &= K \left[\frac{\alpha}{(4\pi)^4 \sin \alpha \cos^2 \alpha} \left(\frac{a\lambda}{\sigma^2} \right)^2 + \frac{\alpha \sin \alpha}{2} \left(\frac{\sigma}{\lambda} \right)^2 \left(\frac{K}{K'^2} - \frac{K}{K''^2} \right) \right] \end{aligned} \quad \text{(III-29)}$$

This result indicates an increase of the reflected mean power with the increase of the incident angle. Such a behavior checks with the experimental result of Taylor [1964].

3.4 Comparison with experiments

In Figure 3, 4, and 5 curves are plotted using (III-28) for comparison with the experimental results of moon returns obtained by Evans and Pettengill [1963] and Lynn et. al. [1964]. The experimental

curves of Evans and Pettengill are obtained using circular polarization and the results given here are for vertical polarization. Thus, the comparison is not meaningful when the angle of incidence is too large, say, over 45° . Comparison is also made with earth data obtained by Taylor [1959] in Figure 6, Dye [1959] and MacDonald [1956] in Figure 7. It is seen that there is a very definite improvement over the works of Daniels [1961], Hayre [1961] and Hughes [1962]. This is due to the contribution of the second and other terms which prevent the too rapid drop off at angles of incidence from about 20° on.

It is interesting to observe that by combining the integrated results of the first two terms in (III-28) approximating D' by D , we obtain a term of the form (Appendix 6)

$$\bar{P} = K \frac{1}{(4\pi)^2} \frac{\alpha (1 + \sin^2 \alpha)}{\sin \alpha \cos^4 \alpha} A \left[1 + A \frac{\sin^2 \alpha}{\cos^4 \alpha} \right]^{-3/2},$$

where $A = \left[\frac{a \lambda}{4 \pi \sigma^2} \right]^2$.

This term has the similar behavior as the results of Beckmann [1963] and Hagfors [1964] for that range of α for which $\sin^2 \alpha \ll 1$. It is important to note that the rest of the terms in (III-28) are not negligible when $\alpha \gtrsim 30^\circ$ (see Figures 3-7).

In all the Figures crosses will be used to indicate the final theoretical results and circles to indicate the theoretical results with contributions from terms higher than the first derivative ignored. Parameter values are tabulated in Table I.

TABLE I
PARAMETERS TO FIT VARIOUS DATA

EXPERIMENTER	TYPE OF TERRAIN	WAVELENGTH (cm)	$A = \left[\frac{L \lambda}{4 \pi \sigma^2} \right]^2$
Evans	Moon	3.6	20
Pettengill	Moon	68	133
Lynn et. al.	Moon	0.86	3
Taylor	Smooth Concrete	Ka	13
Taylor	Smooth Asphalt	x	5
Dye	Ocean	x	900
Macdonald	Ocean	24	20

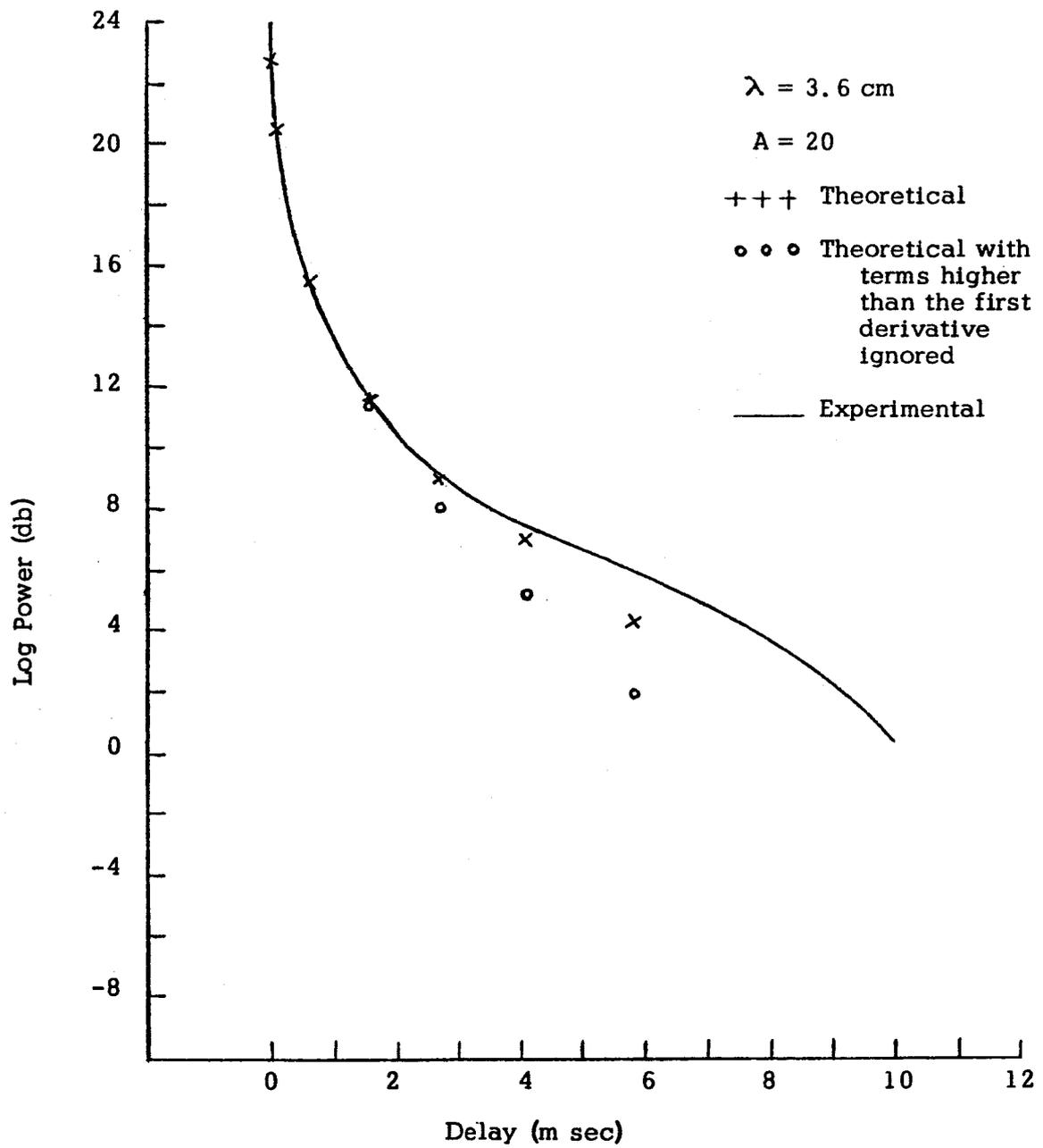


FIGURE III-3

COMPARISON WITH LUNAR RETURNS

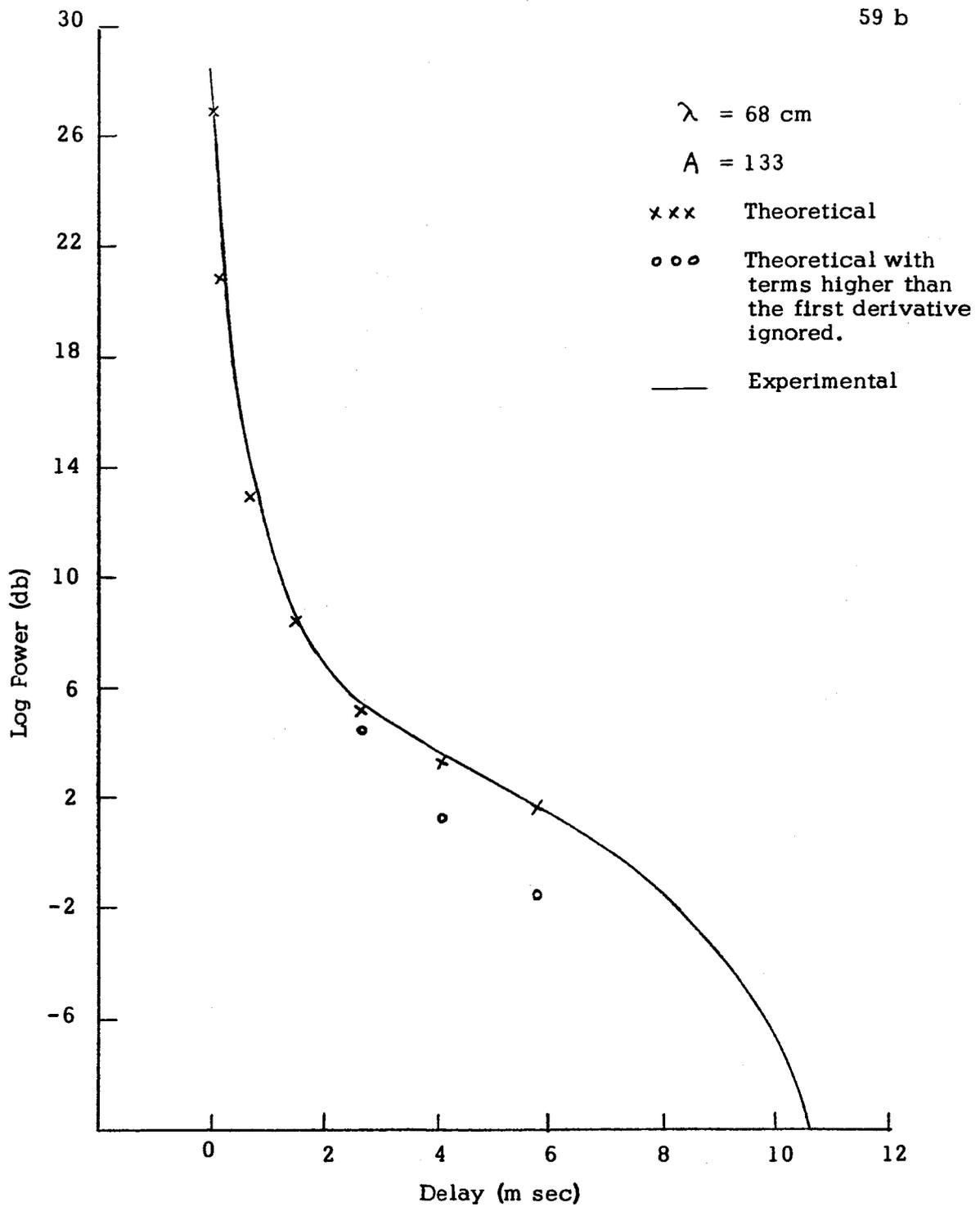
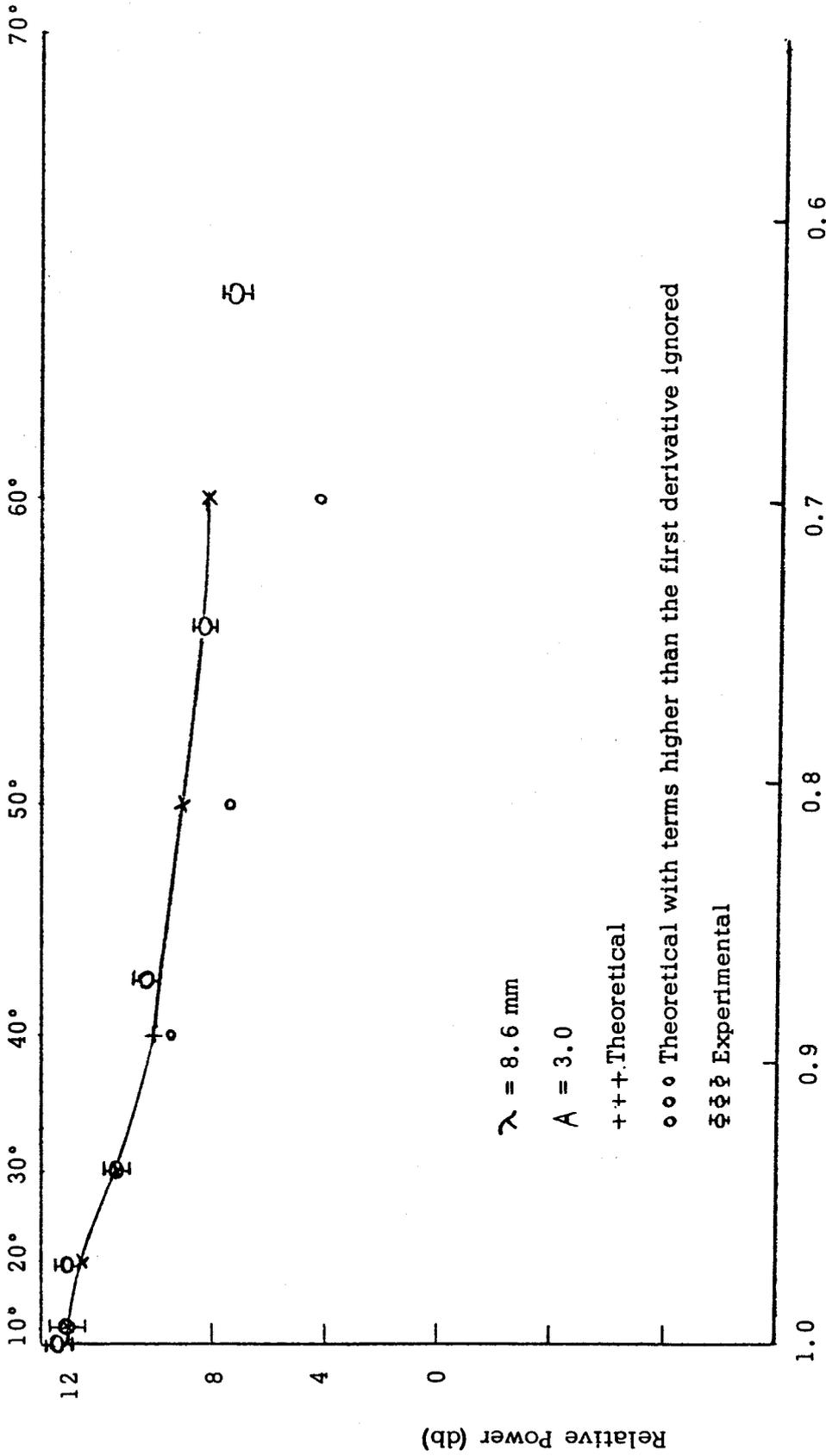


FIGURE III-4
COMPARISON WITH LUNAR RETURNS



$i + \text{Log } \cos \alpha$

FIGURE III-5

COMPARISON WITH LUNAR RETURNS

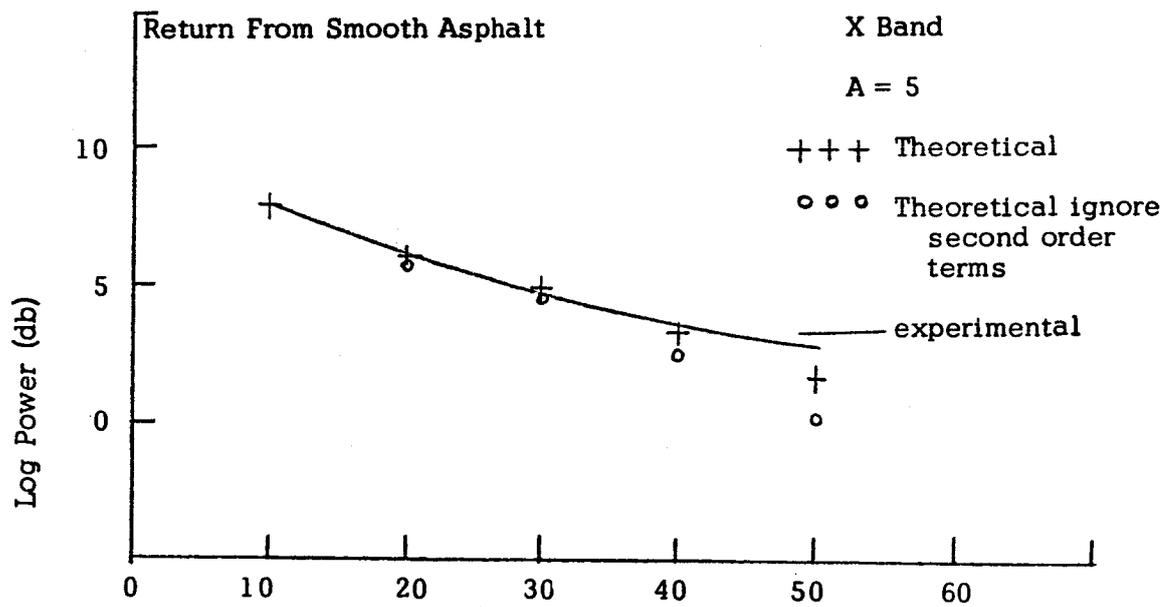
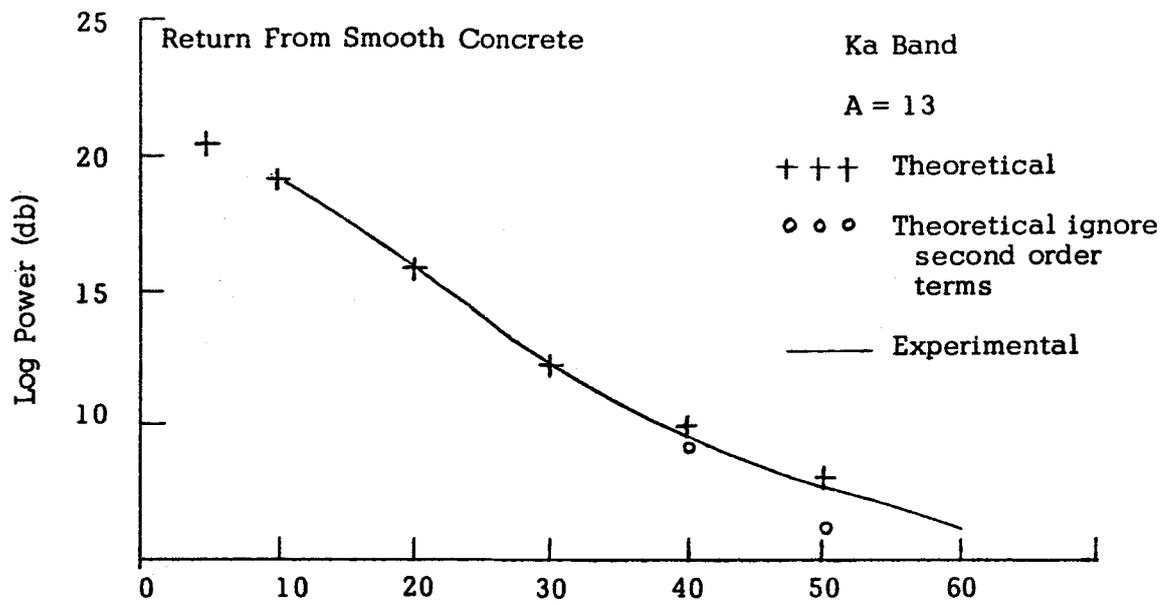


FIGURE III-6

COMPARISON WITH EARTH RETURNS

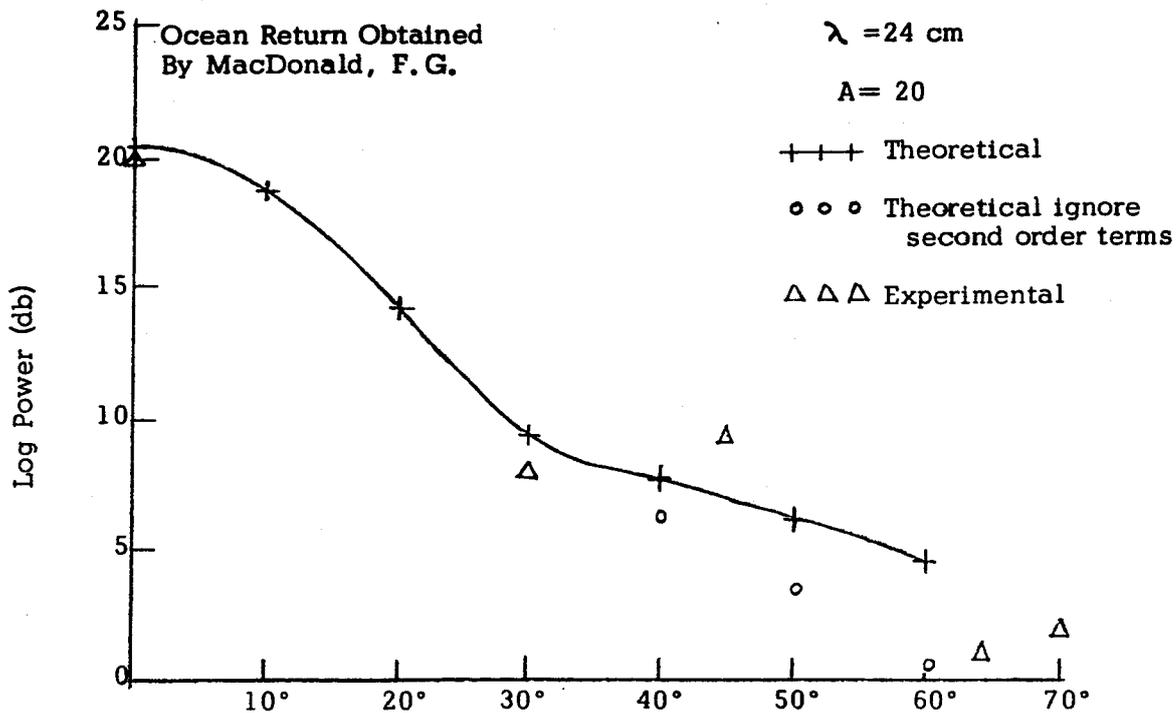
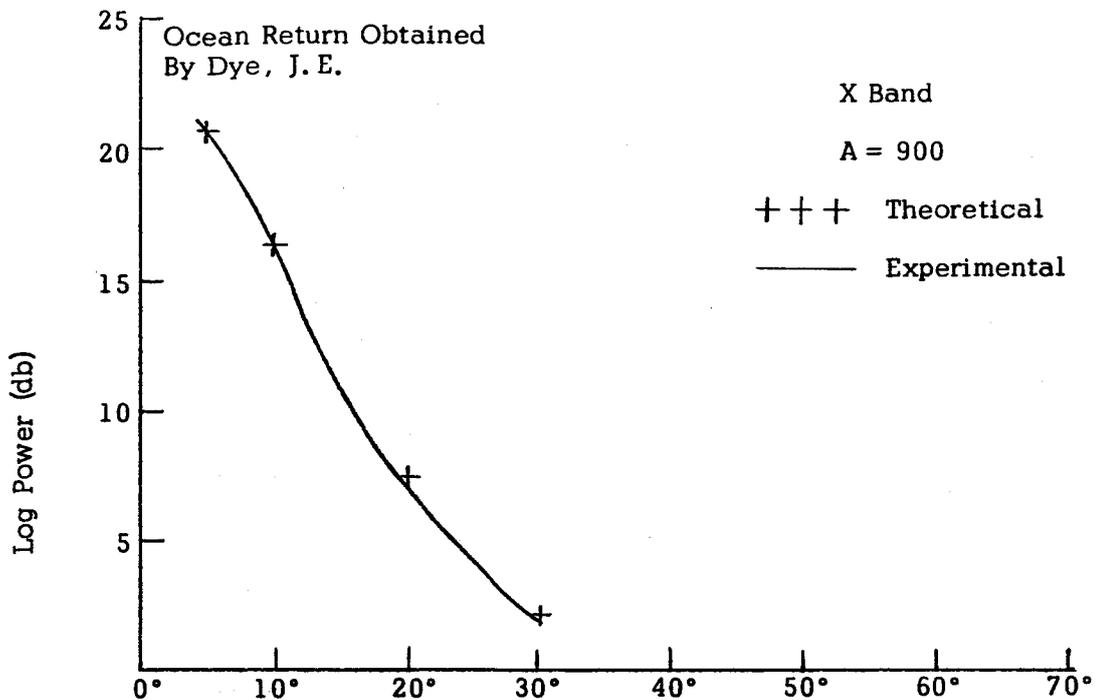


FIGURE III-7

COMPARISON WITH OCEAN RETURNS

CHAPTER IV

SOME PROBLEMS ABOUT THE RADAR SCATTER THEORY

4.1 Introduction

The results of the comparison between the theory and the various experiments in the previous Chapter bring about a number of questions. First of all, the statistical parameters of the surface appear to have a frequency dependence. Experimental investigation also bears out this fact [Evans 1962]. The conclusion arrived at by the experiment is that radar measurements yield information only about the presence of irregularities on the surface that have sizes ranging from less than one to tens of wavelengths. Structures which are considerably smaller than the wavelength may never be detected and large structures may be examined only if they are not covered by smaller irregularities. A general theoretical proof of the above conclusion is as yet lacking, but for cases where Kirchhoff's approximation applies it is possible to show that these measured statistical parameters are, indeed, frequency dependent and characterize irregularities only of sizes seen at the given frequency. The true statistical parameters of the surface are obtainable only when the exploring wavelength is about four times larger than the standard deviation of the surface. Detailed discussions on the frequency dependent property will be found in the next section.

As a whole, the moon does not have a uniform distribution of structure sizes in all directions. Thus, it is questionable whether or not the fitting of the moon return in the previous Chapter has any meaning. If, however, the exploring wave has sampling filter effect, then the result of curve fitting may still be meaningful, since this requires structures of some instead of all sizes to be uniformly distributed over the moon's surface.

Still another observation that should be made from previous results is that the fitting of curves gets bad in general after about 35° . Many reasons are, of course, possible; it may be due to inadequate description of the surface autocorrelation function; it may be due to some shadowing and multiple reflections, or maybe it is because of the depolarization and polarization effects in the case of moon returns and

perhaps imperfect conductivity of the surface. However, it is known that the roughness of a surface modifies the scattered field far more than its electrical properties [Beckmann and Spizzichino 1963]. The effects of shadowing and multiple reflections should still be negligibly small up to 50° for relatively flat surface [Beckmann 1964]. The same is true of polarization [Ament 1960] and depolarization [Evans and Pettengill 1963]. Thus, the effect is most likely due to the use of an autocorrelation function which does not describe adequately the surface in question. A discussion on this problem will be found in section 4.3, where we see that a more adequate autocorrelation function of the surface does lead to a very close fit from near vertical to near grazing.

Since the purpose of investigating radar returns is, in this case, to learn about the surface structure sizes, having an exact theory that is too involved and, consequently, non-informative is not a desirable solution. On the other hand, an approximate theory that takes care only of main contributions and is able to provide useful information about the surface may very well be more desirable. Since a random surface is characterized by its probability distributions and autocorrelation function of surface heights, the use of appropriate functions becomes essential.

Besides the questions mentioned above, the size of the illuminated area also presents a problem. It is clear that while structures large compared with the dimensions of the illuminated area cannot have significant effect on the mean return power, large undulations of sizes comparable in dimension to the illuminated area will certainly have some effect. In the case of backscattering, the effect will be seen to give rise to a lower mean return power. Detailed discussion on this problem will be given in section 4.5.

4.2 The problem of frequency dependence of the measured statistical parameters of the surface.

Many authors [Daniels 1961; Hayre 1961; Winter 1962; Hughes 1962; Fung and Moore 1964] have treated the rough surface scattering problem as a statistical one and employed the Kirchhoff-Huygens Principle to obtain an approximate expression for the mean return power. Attempts were also made to fit the moon and the earth data to determine

the statistical parameters for the rough surfaces [Hayre 1961, 1963; Evans and Pettengill 1963; Muhleman 1964]. It turns out that the numbers obtained for the correlation distance and the standard deviation of the surface heights or the rms slopes are different at different frequencies. The question, therefore, arises as to the meaning of these numbers and their relations, if any, to the true statistical parameters of the surfaces. In what follows, we restrict our considerations to surfaces with Gaussian distribution of surface heights which are characterized by monotone decreasing surface correlation functions. We also restrict ourselves to cases where Kirchhoff's approximation [Beckmann and Spizzichino 1963] applies.

4.2.1 The effective parameters and their significance

The expression for backscattered angular power obtained by Beckmann [1963] is

$$P(\theta) \propto \cos^2 \theta \int_0^D J_0(2k\xi\theta) \xi \exp[-K(1-\rho(\xi))] d\xi \quad (IV-1)$$

where $K = 16\pi^2 (\sigma/\lambda)^2 \cos^2 \theta$

λ = wavelength of the incident radiation

$$k = 2\pi/\lambda$$

D = half the radius of the illuminated area

$\rho(\xi)$ = surface autocorrelation coefficient with which is associated a correlation distance, l .

Consider now the case of near vertical incidence. [For a discussion of the relation between structure sizes and larger angles of incidence from the vertical see Fung and Moore, 1964]. Then the value of K depends mainly on the ratio of σ to λ . For a given surface K depends then on λ alone. If the value of K is big due to small λ , $\rho(\xi)$ cannot deviate from unity very much before $\exp[-K(1-\rho(\xi))]$ becomes so small that integration over larger

values of ξ gives negligible contribution to the integral. In general, for any given small fixed ϵ , we can find corresponding to a given frequency a ξ_1 , $0 < \xi_1 < D$, such that

$$\int_0^D J_0(2k\xi\theta) \xi \exp[-K(1-p(\xi))] d\xi = \int_0^{\xi_1} J_0(2k\xi\theta) \xi e^{-K[1-p(\xi)]} d\xi + \epsilon$$

Observe that the value of ξ_1 , as defined above varies as frequency varies, since K is a function of frequency. If ϵ is chosen small enough, we can write

$$\int_0^D J_0(2k\xi\theta) \xi \exp[-K(1-p(\xi))] d\xi \approx \int_0^{\xi_1} J_0(2k\xi\theta) \xi e^{-K[1-p(\xi)]} d\xi \quad (\text{IV-2})$$

Now, let $\eta = 1 - p(\xi)$. Then η depends on λ because ξ_1 does. Let us define $p_1(\xi)$, the effective correlation coefficient, as

$$\eta p_1(\xi) + (1-\eta) = p(\xi), \quad 0 \leq \xi \leq \xi_1 \quad (\text{IV-3})$$

Substituting (IV-3) in (IV-2), we have

$$\begin{aligned} & \int_0^D J_0(2k\xi\theta) \xi e^{-K[1-p(\xi)]} d\xi \\ & \approx \int_0^{\xi_1} J_0(2k\xi\theta) \xi \exp[-K(1-\eta p_1(\xi)) - 1 + \eta] d\xi \\ & = \int_0^{\xi_1} J_0(2k\xi\theta) \xi \exp[-K\eta(1-p_1(\xi))] d\xi \\ & = \int_0^{\xi_1} J_0(2k\xi\theta) \xi \exp[-K_1(1-p_1(\xi))] d\xi \quad (\text{IV-4}) \end{aligned}$$

where $K_1 = 16\pi^2(\sigma'/\lambda)^2 \cos^2\theta$

$\sigma' = \sigma\sqrt{\eta}$, the effective standard deviation of the surface.

Since σ of the surface is a fixed number, σ' depends on frequency the same way $\sqrt{\eta}$ does. At the frequency which allow (IV-2) to hold, the mean power return curve obtained from the experiment is thus seen to result by (IV-4) from an effective surface defined by σ' and $\rho_1(\xi)$. If λ' is the correlation distance associated with $\rho_1(\xi)$, then σ' and λ' are the statistical parameters obtainable from fitting the experimental curve. This must be so since two points on the surface that are farther apart than ξ_1 could not be distinguished. This then places an upper limit on the structure size that can be observed at this frequency. Thus, when Hayre [1961] obtained a good fit of the moon data with $\lambda = \lambda$ and $\sigma = 0.1 \lambda$ at $\lambda = 68$ cm, it is actually λ' and σ' which he obtained.

4.2.2 Estimate of σ'

Consider the factor, $\exp[-K_1(1 - \rho_1(\xi))]$ which is unity at $\xi = 0$ and down to, say, $\exp[-b]$ at $\xi = \xi_1$, where b is a positive real number and ξ_1 has the same significance as defined previously so that $\rho_1(\xi_1) = 0$. Hence, at $\xi = \xi_1$,

$$K_1 = \eta K = b$$

$$\eta = \frac{b}{K} = \frac{b}{(4\pi\sigma/\lambda)^2 \cos^2\theta}$$

Since $\eta = (\sigma'/\sigma)^2$, we get by comparison

$$\sigma' = \sqrt{b} \lambda / (4\pi \cos\theta) \quad (\text{IV-5})$$

If a reasonable range of the values of $\exp[-b]$ is between 2.5×10^{-3} and 3.3×10^{-4} corresponding to $6 \leq b \leq 8$, then at near vertical incidence, the order of magnitude of σ' in terms of λ is

$$\sigma' \approx 0.21 \lambda \quad (\text{IV-6})$$

The above relation together with λ' associated with $\rho_1(\xi)$ gives an indication as to what range of structure sizes are being seen at a given frequency, i. e. it shows the frequency sampling effect on surface structures. It also explains why a small σ to λ ratio will fit the

experimental curve while the actual σ of the surface may, in fact, be much larger than λ .

From the discussions above, we conclude that in general, the statistical parameters obtained by fitting the experimental curves are the effective parameters and they do not equal to the actual parameters of the surface. They characterize the portion of the structures on the surface that have been seen at the given frequency. The effective standard deviation of the surface will, however, coincide with the actual one when the wavelength used is of the order of about four times the actual standard deviation of the surface. To illustrate the above ideas, let us consider the moon data at $\lambda = 68$ cm. The best fit to the angular power return curve using only the zero order term in the mean return power expression gives a value of 110 for the parameter, $[\lambda \ell / (4\pi \sigma^2)]^2$ [Evans and Pettengill 1963]. This leads to the relation

$$\ell = 194 \sigma^2 \quad \text{meters} \quad (\text{IV-7})$$

The result of this paper shows that the relation should be

$$\ell' = 194 \sigma'^2 \quad (\text{IV-8})$$

Using (IV-6) we get

$$\ell' \approx 194 (0.21 \times 0.68)^2 = 3.96 \quad \text{meters}$$

If, on the other hand, one believes that ℓ and σ are not effective values, then the following result is obtained for ℓ when a reasonable value of 1000 m is used for σ of the moon,

$$\ell = 194 (1000)^2 = 194000 \quad \text{km}$$

This value of ℓ is larger than the circumference of the moon!

4.3 The problem of angular dependence of the mean return power and surface autocorrelation function.

Radar returns from terrestrial and lunar rough surfaces have been explained in part by many theories. As yet, however, there is no theory that can explain satisfactorily the variation with angle of incidence of the observed signals over the entire range from normal to near grazing, although various attempts have been made in this direction [Muhleman 1964; Beckmann 1964]. As we mentioned before there are many factors that effect the return power at large angles of incidence. Up to the moment each explanation is given in terms of only one factor; Beckmann [1964] considered shadowing effect and Muhleman [1964] assumed the existence of effective slopes. A rigorous theory that takes into account the vector nature of the wave, the depolarization effect due to rough surface scattering, the shadowing and multiple reflections and the inhomogeneity and imperfect conductivity of the surface is definitely lacking. However, it may not be desirable to have such a theory unless it can provide us with more useful information about the surface and in a practical way. With this view in mind, we concentrate on the question of proper description of the surface roughness and try to obtain an approximate result that takes care only the dominating returns.

Our previous discussion has shown that the effective surface-height autocorrelation function is wavelength dependent, although the actual function is of course a property of the surface alone. Various observations also indicate that scattering behavior of rough surfaces has a wavelength variation of λ^2 to λ^4 [Janza 1963]. Part of this wide range is due to differing types of wavelength variation at different angles with the vertical. Part of the variation quoted for experiment is undoubtedly due to nonidentity of the illuminated areas and to experimental difficulties. Another point that needs to be made before we can arrive at an effective surface -- height autocorrelation function is that the effective standard deviation of the surface is angular dependent. To see this, consider the power returned to Q due to an incident spherical wave (see Figure IV-1). From the work of Davies [1954], the power is given by

$$P_r = \frac{P_t G A_p}{4\pi\lambda^2 R_0^2} \cot^2 \alpha \iint \exp\left\{\frac{4\pi i}{\lambda}[R - Z(R, \theta) \cos \alpha]\right\} dR d\theta$$

$$\cdot \iint \exp\left\{-\frac{4\pi i}{\lambda}[R' - Z(R', \theta') \cos \alpha']\right\} dR' d\theta'$$

(IV-9)

where A_p is the receiving antenna aperture
 P_t is the power transmitted
 G is the antenna gain
 R is the slant height
 λ is the wavelength of the transmitted wave
 Z is the random function of position denoting the height of the surface

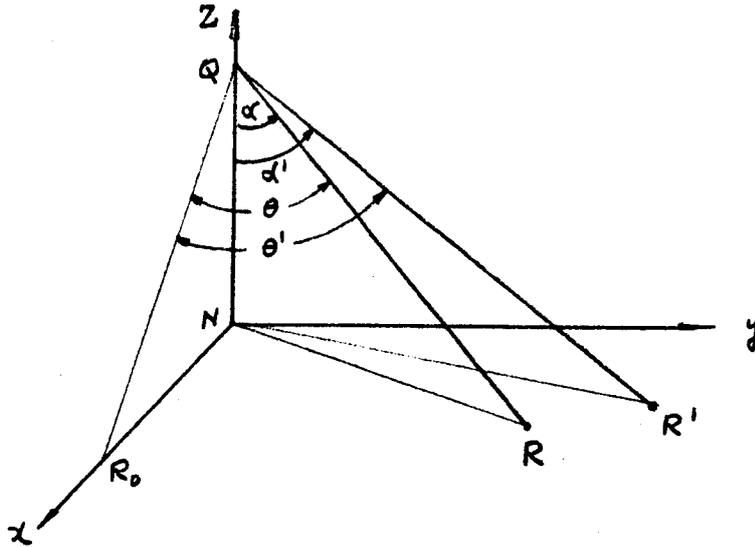


Figure IV-1
Disposition of radar and surface

Let $R' = R + t$ and $\theta' = \theta + \varphi$. Then (IV-9) becomes

$$P_r = \frac{P_t G A_p}{4\pi\lambda^2 R_0^2} \cos^2\alpha \iiint \exp\left[-\frac{4\pi i}{\lambda} t\right] \exp\left\{-\frac{4\pi i}{\lambda} [Z(R, \theta) \cos\alpha - Z(R+t, \theta+\varphi) \cos\alpha']\right\} dR d\theta dt d\varphi \quad (\text{IV-10})$$

To find the mean power, the quantity

$$\exp\left\{(-4\pi i/\lambda)[Z(R, \theta) \cos\alpha - Z(R+t, \theta+\varphi) \cos\alpha']\right\} \quad (\text{IV-11})$$

must be averaged with respect to an appropriate density function, which is usually assumed to be Gaussian. It is important to observe that the quantity to be averaged is given by (IV-11) and is not

$$\exp\left\{(-4\pi i/\lambda)[Z(R, \theta) - Z(R+t, \theta+\varphi)] \cos\alpha\right\} \quad (\text{IV-12})$$

the expression used by Davies and others. However, when the correlation distance is small compared with the distance required for significant variation in $\cos\alpha$, (IV-12) is a good approximation; i. e. it is reasonable to approximate $\cos\alpha'$ by $\cos\alpha$. The average of (IV-12) with respect to a Gaussian joint probability density is then given by

$$\exp\left\{(-4\pi/\lambda)^2 \sigma^2 \cos^2\alpha (1 - \rho)\right\} \quad (\text{IV-13})$$

where σ is the standard deviation of the surface heights and ρ is the associated correlation coefficient.

Since the variables involved in averaging the phase term of (IV-11) are $Z \cos\alpha$ and $Z' \cos\alpha'$, not Z and Z' , the product $\sigma \cos\alpha$ must be considered as a single quantity rather than as the product of two unrelated quantities. We define this as the effective standard deviation of heights about the mean,

$$\sigma' = \sigma \cos\alpha \quad (\text{IV-14})$$

The autocorrelation function must also be defined in terms of the effective heights $Z \cos \alpha$ and $Z' \cos \alpha'$. By definition it is therefore

$$\overline{Z \cos \alpha Z' \cos \alpha'} \quad (IV-15)$$

where the bar denotes the ensemble average. As with the averaging process involved in determining (IV-13), it is often possible to consider $\cos \alpha$ as essentially constant over the region of correlation, so that the local average involved in (IV-15) is given by

$$\overline{Z \cos \alpha Z' \cos \alpha'} = \overline{Z Z' \cos^2 \alpha}$$

Thus, the radar return is determined by an effective height above the mean surface and its statistical parameters, not by the actual height and its statistics. The effective height includes both the properties of the surface and a parameter of the experiment, the angle of illumination. For rough surfaces phase coherence of the signal is lost over a sufficiently short distance so that the bias factor $\cos \alpha$ may be considered to be the same for all elements of the population involved in the local region over which the averages must be performed. Hence, the averages performed involve the height as the random variable, with the $\cos \alpha$ as a constant multiplier.

Application of the theory often involves consideration of returns from a wide range of angles, either separately or as elements of a power superposition of random contributions from different angles. In such considerations, the fact that all measures of height must be multiplied by $\cos \alpha$ must not be ignored. It is unreasonable to expect a correlation coefficient for Z alone to be effective in describing the ground in such a theory. In the next section a correlation coefficient is postulated in which the effect of the $\cos \alpha$ is taken into account.

4.3.1 The proposed effective correlation coefficient.

Properties of the surface enter the power return expression only through the correlation function and the standard deviation. Thus, any attempt to determine the properties of a surface that will return a

mean scattered power must be concentrated on these two quantities. The standard deviation is a single number to be determined, whereas ρ is a function. Thus, a form must be assumed (or determined) for this function, and parameters of the function ascertained so that the theory gives the desired form for the return.

Previously postulated correlation functions have involved simple one- or two-parameter expressions. Because of the wide variability obtained from most measurements of the earth performed from aircraft, this seemed accurate enough, although no one has claimed to have a function that agrees with observed variation of scattering with angle of incidence over a wide range. The usual statement is that different theories are called for in different ranges of angle of incidence.

A popular correlation function, which fits the function obtained along simple contours on terrestrial maps reasonably closely, is the exponential $\rho = \exp[-\xi/L]$. This is a one-parameter function, for the only parameter is the correlation length. If the correlation length L is large, the surface structure is presumed to be large. If the ratio L/σ is large, the slopes are small, and the return is much stronger near the vertical than at angles of, say, 30° . If the correlation distance is small, the structure is small, and the signal is weaker at normal incidence than for large structure but stronger at large angles with the normal than for large structure.

It has been observed in many experiments that the value of correlation distance L that gives a good fit to the scattering curve measured near-normal results in a theoretical scattered signal at middle incidence angles that is much weaker than the observed signal. The value of L that would give the observed signals at middle angles is much less than that required to fit the observations near normal.

Figure IV-2 shows typical sample exponential autocorrelation functions that could be made to fit the two ranges of an experimental scattering curve. Here the larger correlation distance, which fits near normal, is L , and the smaller one is ℓ . The entire significant contribution of the middle-angle autocorrelation function occurs for small values of ξ so that $\exp[-\xi/L]$ is essentially unity. This

suggests that the choice of an autocorrelation function that will fit observation over a wide range of angles demands careful attention to the shape of the function at its very beginning, for a steeper decline of the correlation for short distances is required to obtain significant returns at middle angles.

Lunar measurements have been made more accurately over a wide range of angles than terrestrial measurements [Evans and Pettengill 1963]. The rapid decrease of the initial part of the return clearly suggests that this part is due to relatively large, flat facets. At a later time delay the return is slowly decreasing, which implies that the surface appears to be rougher or the contribution is from smaller structures. At a still later time, we expect only structures with significant slopes to contribute significantly, and these are likely to be still smaller. Thus, the return curve suggests, in accord with the above discussion, at least three different sizes of scatterer. A correlation function that behaves for small lag distances like one for a very rough surface, and behaves like one for a somewhat smoother surface with larger structure, and finally behaves like one for a surface with large, relatively flat structures is called for.

Returns at large angles from small structures alone would be due to an exponential correlation function having a very small correlation distance. The intermediate returns from intermediate angles call for an exponential with intermediate correlation distance. The large returns at small angles call for a large correlation distance.

The following correlation coefficient is suggested to account for these various sizes of structures. In fact, it has four components, accounting roughly for the angular ranges 0° to 20° , 20° to 50° , 50° to 70° and 70° to 80° . For motivation to the form of $\rho(\xi)$, see Appendix 7.

$$\begin{aligned} \rho(\xi) = & 1 + K^{-1} \ln \left\{ (c/a) \exp[-K(1 - e^{-|\xi|/L})] \right. \\ & + \frac{d}{a} \exp[-K(1 - e^{-|\xi|/l})] + \frac{f}{a} \exp[-K(1 - e^{-|\xi|/l'})] \\ & \left. + \frac{g}{a} \exp[-K(1 - e^{-|\xi|/l''})] \right\} \end{aligned} \quad (\text{IV-16})$$

where $K = 4k^2\sigma'^2$

$$\xi = (R^2\varphi^2 + t^2 \csc^2 \alpha)^{1/2}$$

σ' = the effective standard deviation of the surface heights.

$$k = 2\pi/\lambda$$

λ = wavelength

$$a = c + d + f + g$$

L, l, l', l'' are the correlation distances of various structures
 c, d, f, g are appropriate constants.

The assumed autocorrelation function may be interpreted in terms of a multi-point fit to a continuous spectrum of sizes of structures on the surface or a description of several discrete sizes of surface structure. The former interpretation seems to be a more reasonable one, although no distinction can be made on the basis of the data.

Let us now write (IV-16) in the form

$$\begin{aligned} \rho(\xi) = & e^{-|\xi|/L} + K^{-1} \ln \left\{ \frac{c}{a} \right. \\ & + \frac{d}{a} \exp \left[K \left(e^{-|\xi|/l} - e^{-|\xi|/L} \right) \right] \\ & + \frac{f}{a} \exp \left[K \left(e^{-|\xi|/l'} - e^{-|\xi|/L} \right) \right] \\ & \left. + \frac{g}{a} \exp \left[K \left(e^{-|\xi|/l''} - e^{-|\xi|/L} \right) \right] \right\} \end{aligned}$$

(IV-17)

From (IV-17) it is seen that the log term is zero both when ξ is zero and when ξ approaches infinity. Further examination shows that the effect of the log term is to cause the $\rho(\xi)$ to decrease

faster for a range of small values of ξ . This, in fact, is a desirable effect, since smaller structures decorrelate faster, and thus they have been taken into account with this correlation coefficient.

A plot of $\rho(\xi)$ with respect to ξ does not show appreciable difference from a plot of $\exp[-|\xi|/L]$. However, if we plot $1 - \rho(\xi)$ and compare it with $1 - \exp[-|\xi|/L]$ (see Figure IV-5), we see that the difference is tremendous for small values of ξ . Since the integral in (IV-18) is negligibly small except for small ξ , it is clear that such an autocorrelation function produces quite a change in the return power as compared with a simple exponential autocorrelation function.

4.4 An approximate mean power return expression for backscattering and comparison with experiments.

From (IV-10) and (IV-13), the expression for the mean return power is

$$P_r = \frac{P_t G A_p}{4\pi\lambda^2 R_0^2} \cot^2 \alpha \iiint \exp\left[-\frac{4\pi i}{\lambda} t\right] \exp\left[-\left(\frac{4\pi}{\lambda}\right)^2 \sigma^2 (1-\rho)\right] dR d\theta dt d\varphi \quad (\text{IV-18})$$

where the limits of integration are as follows [Davies 1954]

$$\begin{aligned} R_0 - cT/2 &\leq R \leq R_0 \\ -\beta/2 &\leq \theta \leq \beta/2 \\ -\infty &\leq t \leq \infty \\ -\infty &\leq \varphi \leq \infty \end{aligned}$$

(IV-19)

Making use of the fact that σ for the moon is large, we obtain the integrated result to be

$$\begin{aligned}
 P_r = K \frac{\alpha}{a \cos^2 \alpha \sin \alpha} & \left\{ c A_0 \left[1 + A_0 \frac{\sin^2 \alpha}{\cos^4 \alpha} \right]^{-3/2} \right. \\
 & + d A_1 \left[1 + A_1 \frac{\sin^2 \alpha}{\cos^4 \alpha} \right]^{-3/2} \\
 & + f A_2 \left[1 + A_2 \frac{\sin^2 \alpha}{\cos^4 \alpha} \right]^{-3/2} \\
 & \left. + g A_3 \left[1 + A_3 \frac{\sin^2 \alpha}{\cos^4 \alpha} \right]^{-3/2} \right\}
 \end{aligned}$$

(IV-20)

where
$$K = \frac{P_r A_p G C T}{64 \pi^2 R_o^3}$$

$$A_0 = L^2 B^2$$

$$A_1 = l^2 B^2$$

$$A_2 = l'^2 B^2$$

$$A_3 = l''^2 B^2$$

$$B^2 = \lambda^2 / [(4\pi)^2 \sigma^4]$$

The theoretical curves obtained from (IV-20) are compared with the experimental results reported by Evans and Pettengill for both

$\lambda = 3.6$ cm and $\lambda = 68$ cm. These are plotted in Figures IV-3 and 4. The values of A_0 , A_1 , A_2 , and A_3 are obtained by trial and error, and so are the constants c , d , f and g . This last set of constants denotes the relative levels of the terms in (IV-20). One of them, therefore, may always be taken to be one. Comparison is

also made with the experimental results of Lynn et. al. [1964] at $\lambda = 8.6$ mm in Figure IV-6, Taylor [1949] in Figure IV-7, Dye [1958] and MacDonald [1956] in Figure IV-8, Grant and Yaplee in Figure IV-9.

It is worthwhile to compare the results obtained here with the results of a single exponential obtained in Chapter III. The theory of this chapter gives better fit in all cases. However, the results of Chapter III are in many cases good enough. The question arises as to why is this so. For a continuous distribution of structure sizes which is approximately linear or close to being linear, it is reasonable that a fairly good result is obtained by a single exponential-approximation to its correlation function. However, if the distribution of structure sizes is discontinuous or continuous but with large variations, then a single exponential cannot be a good approximation to the correlation function at a frequency which is sensitive to the part of the distribution function that possesses either a discontinuity or a large variation. Thus, in Figure IV-7, both theories give pretty good fit at Ka band, but at X band the single exponential theory does not give as good a result. In fact, the value of A that fits the first portion of the experimental curve cannot fit at large angles of incidence from say, 30° on; whereas, an intermediate value of A , the A -value that lies between A , and A_1 of the theory in this chapter, gives better overall fit but poorer at small angles. This indicates that a single exponential is, as expected, an overall approximation to a more complete autocorrelation function. This fact is also clear from other fittings of experimental curves especially those of Grant and Yaplee in Figure IV-9 and the 8.6 mm moon return in Figure IV-6.

Observe that a larger return at large angles of incidence may be interpreted as follows: there are structures of proper size present that are responsible for it; while a smaller return may mean absence or insufficient number of structures of proper size. Now the use of a single exponential function for the autocorrelation function involves a parameter, the correlation distance, which is an average obtained with all sizes of structures taken into account. However, the average is of such a nature that the effect of big structures dominates. Thus, it cannot account very well for small structures when there are a lot of them nor can it indicate their complete absence when there is none.

This explains why a single exponential theory may give a larger or smaller return than the experimental results. It also explains how the more complete autocorrelation function introduced in this chapter helps to give a clearer understanding of the surface structures.

In Table II parameter values for fitting the experimental curves using the theory of this chapter are tabulated. In Table III values of the parameter A from Table I and those of parameter A_0 from Table II are tabulated side by side together with their associated correlation distances for ease of comparison. The correlation distances are calculated under the assumption that the effective standard deviation at a given frequency is $1/4$ the wavelength.

TABLE II
PARAMETERS TO FIT VARIOUS DATA

Experimenter	Terrain Type	Wavelength (cm)	A_0	A_1	A_2	A_3	c	d	f	g
Pettengill	Moon	68	110	0.25	0.011	0.00143	1	0.8	0.9	2
Evans	Moon	3.6	18	0.5	0.011	0.0001	1	120	535	1666
Lynn et. al.	Moon	0.86	1.8	0.05	0.02		1	3.54	1.00	
Taylor	Smooth Concrete	Ka	10	0.14	0.00143		0.81	14	71.5	
Taylor	Smooth Concrete	x	50	0.27			41	160		
Dye	Ocean	x	500				1			
MacDonald	Ocean	24	10	0.14	0.00143		0.81	14	71.5	
Grant et. al.	Sea	3.2	900	0.02			1	40		

TABLE III
 COMPARISON OF PARAMETER VALUES OF A IN TABLE I
 WITH THOSE OF A₀ IN TABLE II
 TOGETHER WITH THE ASSOCIATED CORRELATION DISTANCES

Experimenter	Terrain	Wavelength (cm)	A ₀	A	$L_0 = \frac{\sqrt{A_0} \pi \lambda}{4}$ (cm)	$L = \frac{\sqrt{A} \pi \lambda}{4}$ (cm)
Pettengill	Moon	68	110	133	560	614
Evans	Moon	3.6	18	20	12	12.6
Lynn et. al.	Moon	0.86	1.8	3	0.905	1.16
Taylor	Smooth Concrete	Ka	10	13	2.48	2.8
Dye	Ocean	x	500	900	52.6	70.6
MacDonald	Ocean	24	10	13	59.5	67.7

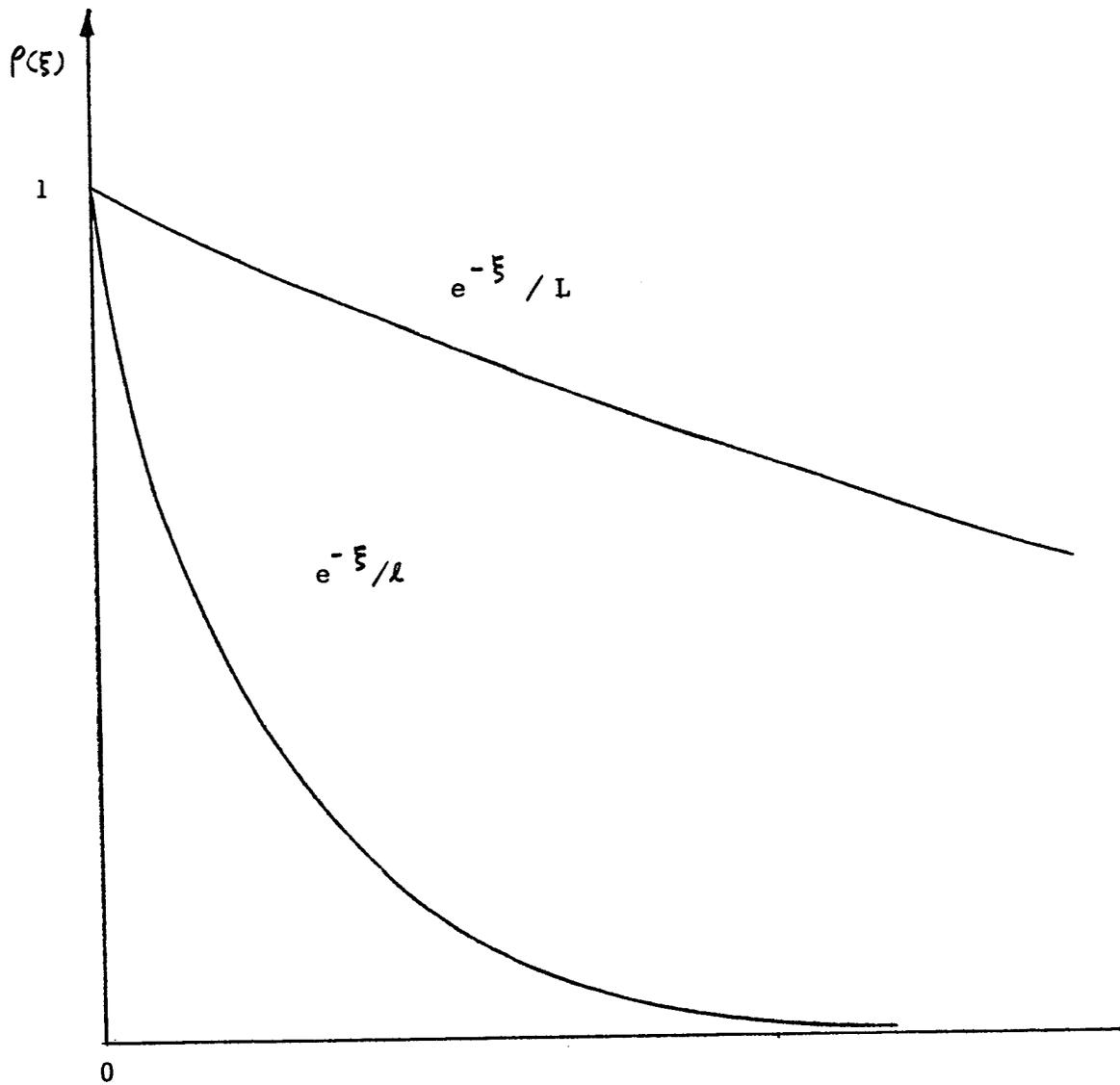


FIGURE IV-2

SAMPLE EXPONENTIAL AUTOCORRELATION FUNCTIONS

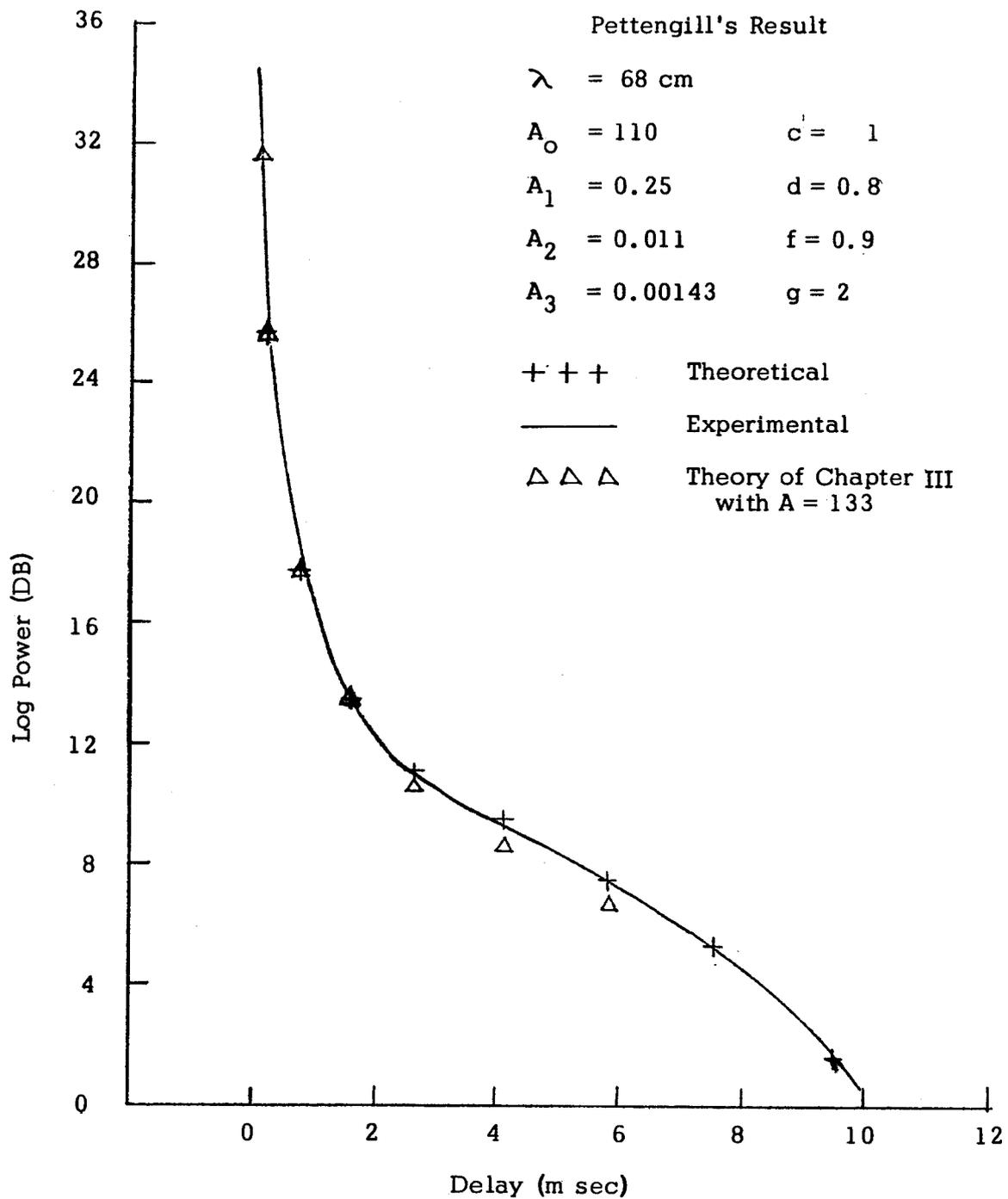


FIGURE IV-3

COMPARISON WITH LUNAR DATA AT $\lambda = 68 \text{ cm}$

$K = 1,000$
 $L = 3,000 \text{ m}$
 $l = 150 \text{ m}$
 $l' = 30 \text{ m}$
 $l'' = 10 \text{ m}$
 $a = 4$
 $c = 1$
 $d = 1$
 $f = 1$
 $g = 1$

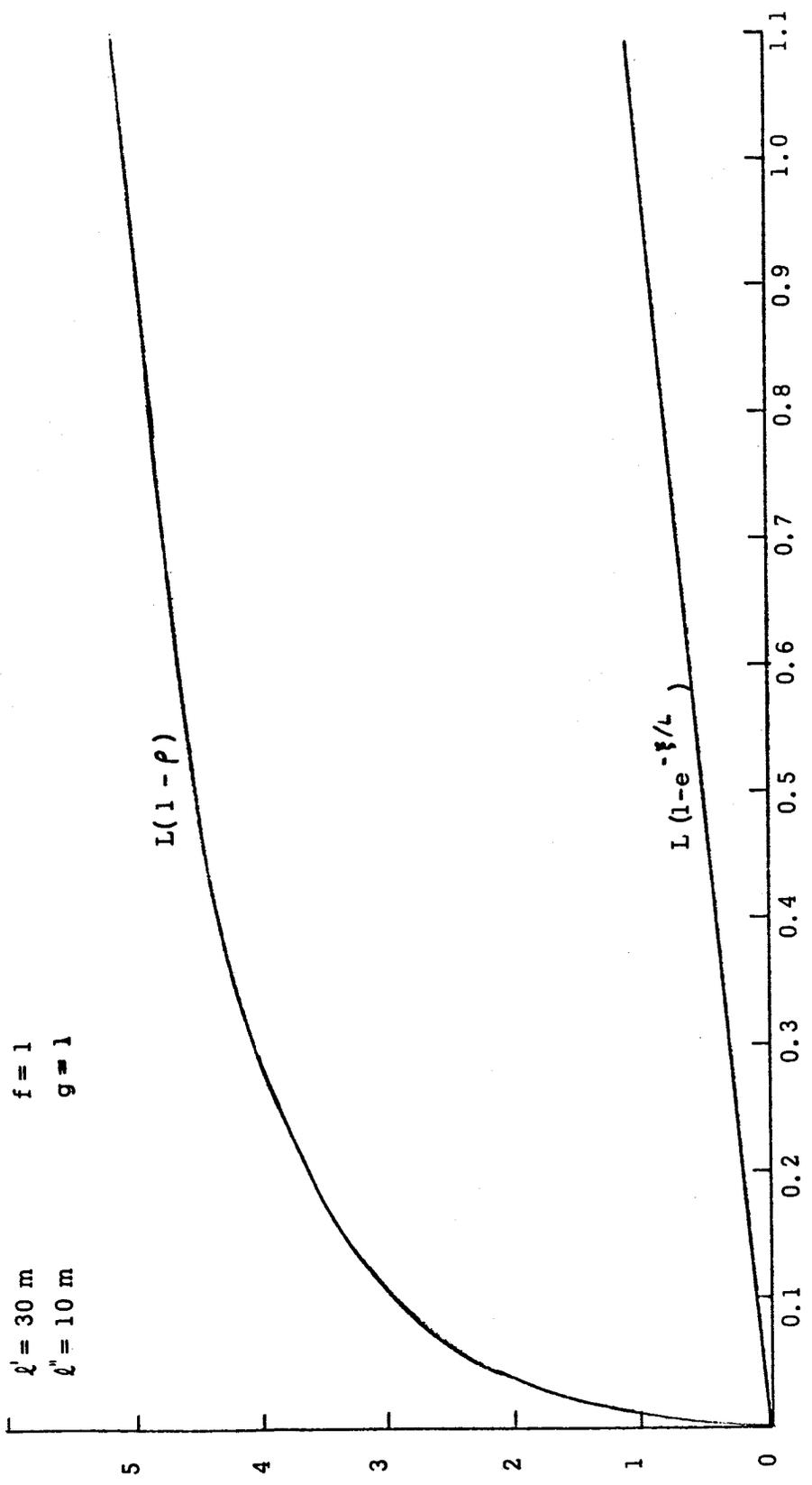


FIGURE IV-5
COMPARISON OF $(1 - \rho)$ WITH $[1 - \text{EXP}(-x/L)]$

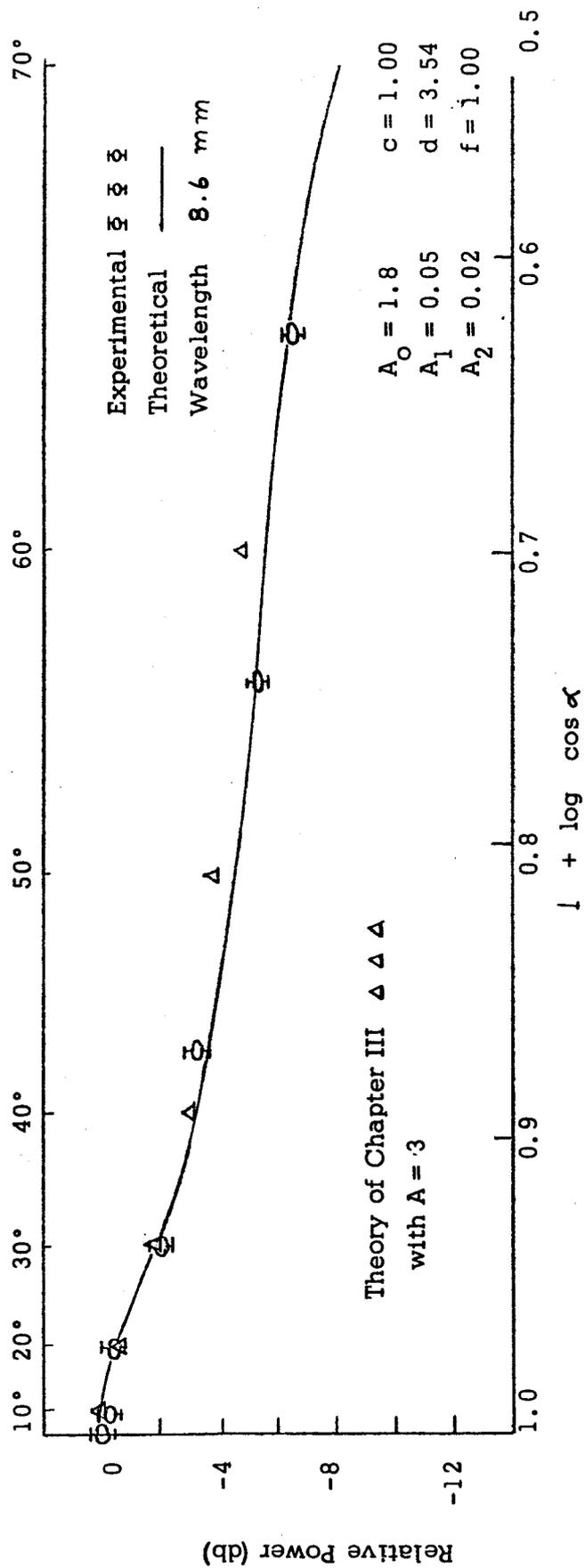


FIGURE IV-6

COMPARISON WITH LUNAR DATA AT $\lambda = 8.6$ mm

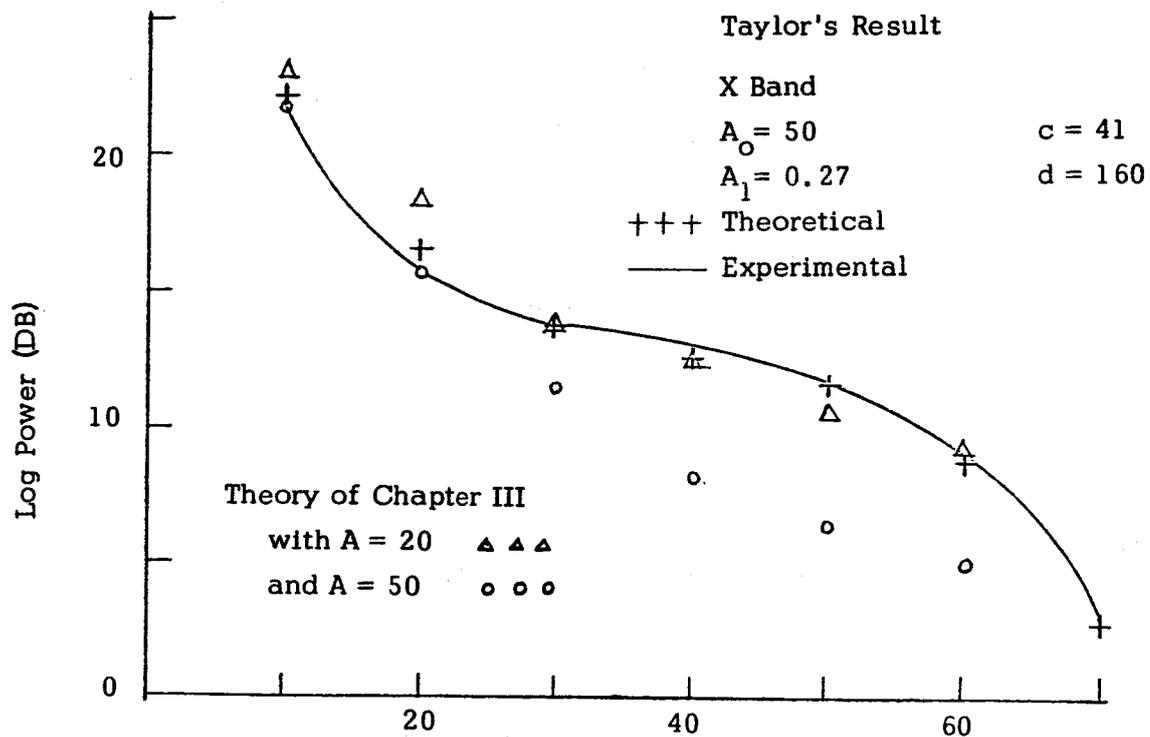
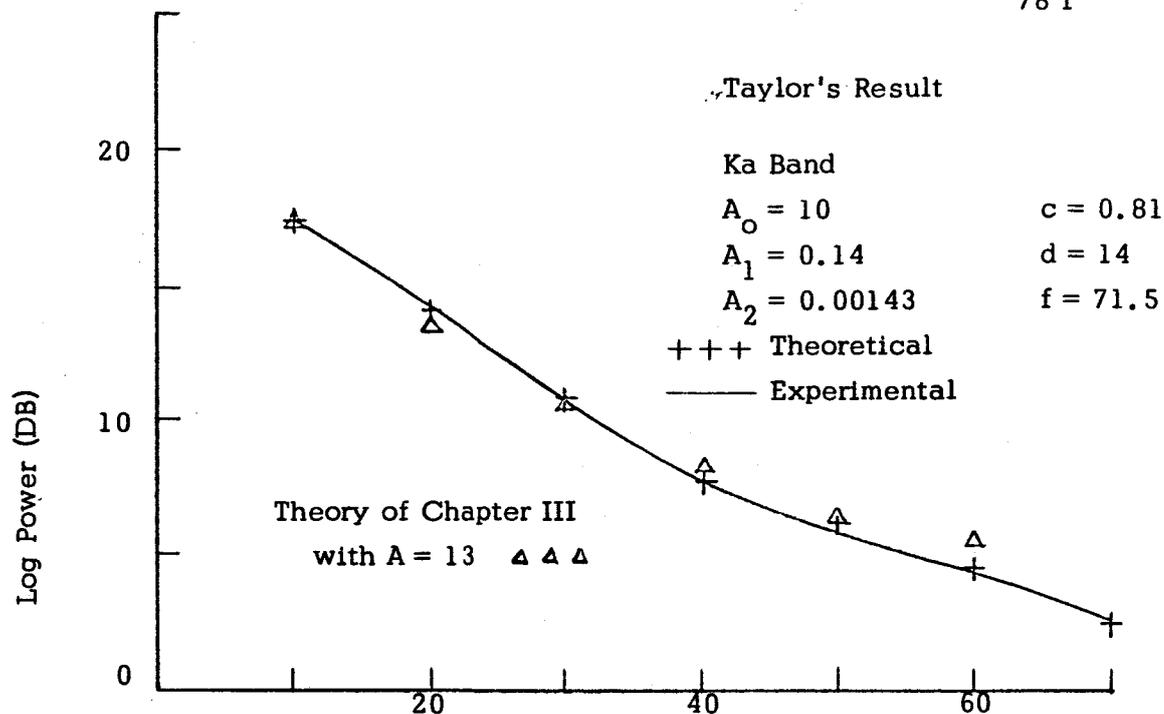


FIGURE IV-7

RETURN FROM SMOOTH CONCRETE
 COMPARISON WITH TAYLOR'S RESULT

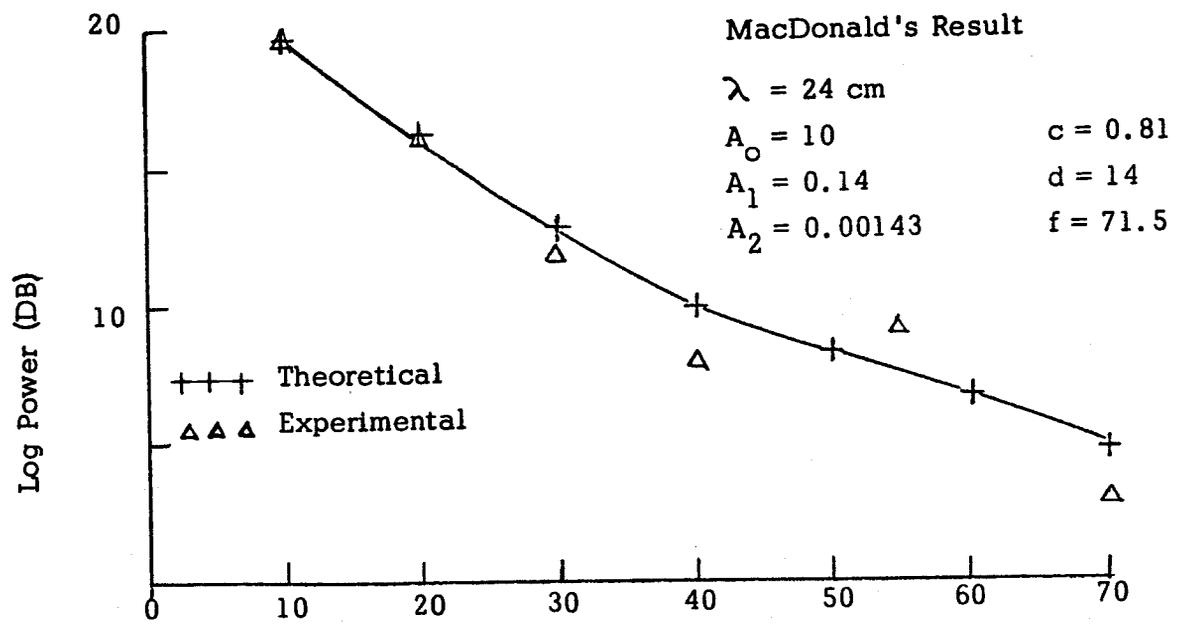
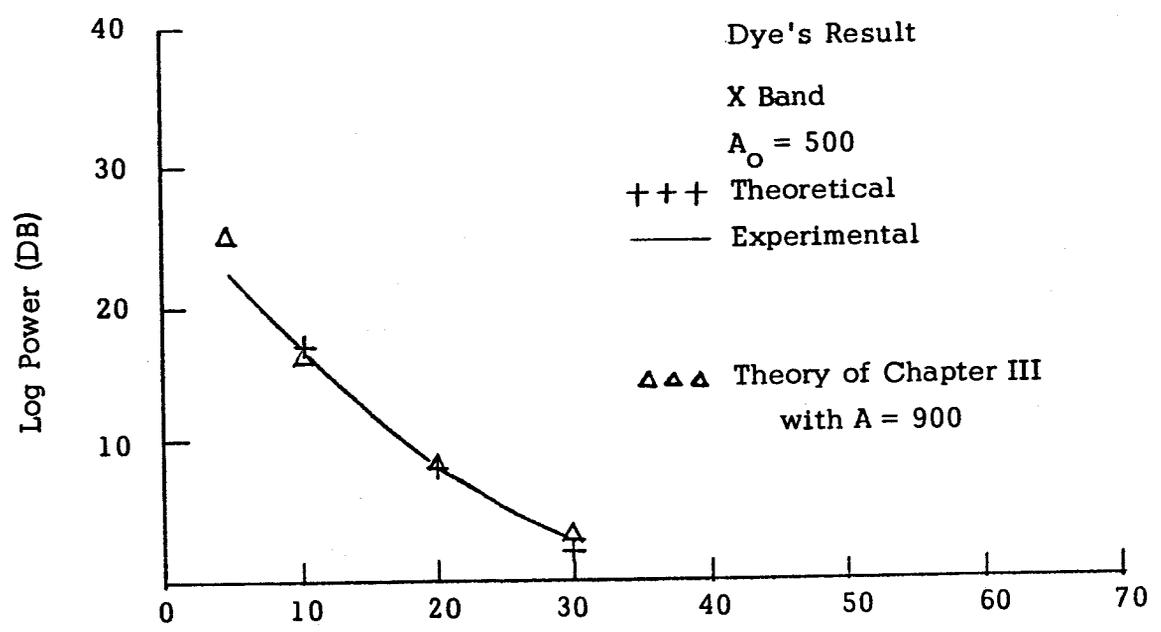


FIGURE IV-8

COMPARISON WITH RESULTS OF DYE AND MACDONALD

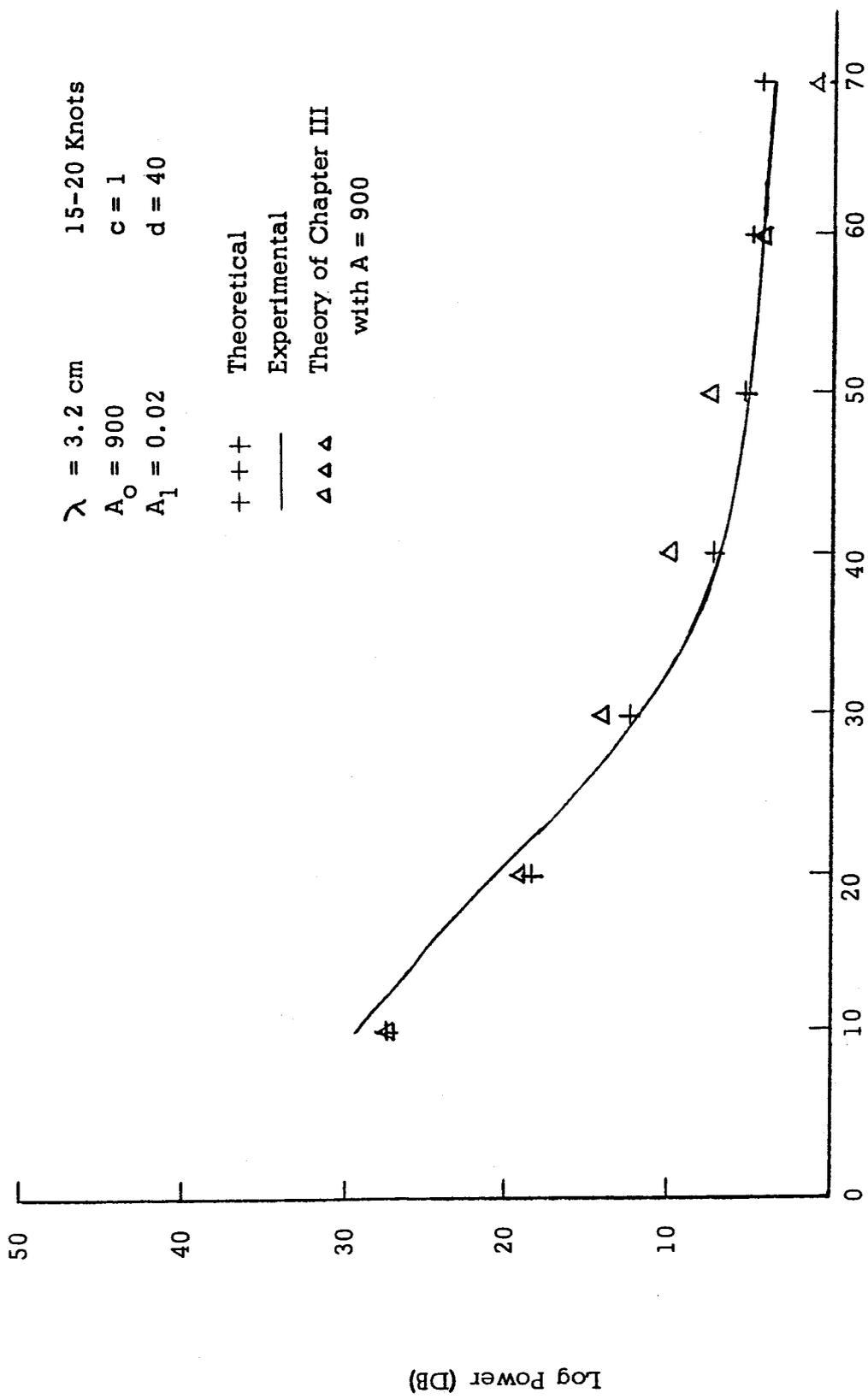


FIGURE IV-9

COMPARISON WITH SEA RETURN OBTAINED BY GRANT AND YAPLEE

4.5 Effect of the size of the illuminated area on radar measurements

Experimental evidence [Evans 1962] shows that when large structures are covered with small structures on top of them, only the small structures can be detected. Using the notion of a composite rough surface Beckmann [1964] showed that if small structures have larger rms slopes, then it is, indeed, the small structures that dominates the scattered return signal. The exact nature as to how the return is affected by large undulations comparable in size to the illuminated area has, however, not been treated. We attempt below to consider the special case of a Gaussian surface characterized by an exponential correlation function of surface heights at near vertical incidence. Consideration is also restricted to backscattering.

The backscattered mean return power due to an incident plane wave is [Beckmann 1963 p. 87]

$$P = K' \cos^{-2} \alpha \int_0^{\infty} J_0(2k\alpha\xi) e^{-K[1-\rho(\xi)]} d\xi \quad (\text{IV-21})$$

where α is the angle of incidence of the incident wave relative to mean ground plane

$$K = (2k\sigma \cos \alpha)^2$$

$$k = 2\pi / \lambda$$

$\rho(\xi)$ = correlation function of the surface

K' = constant of proportionality

Since the large undulations are assumed to be of size comparable to the illuminated area, the mean power given by (IV-21) is obtained as a mean only over the small structures on top of these large undulations (see Figure IV-10).

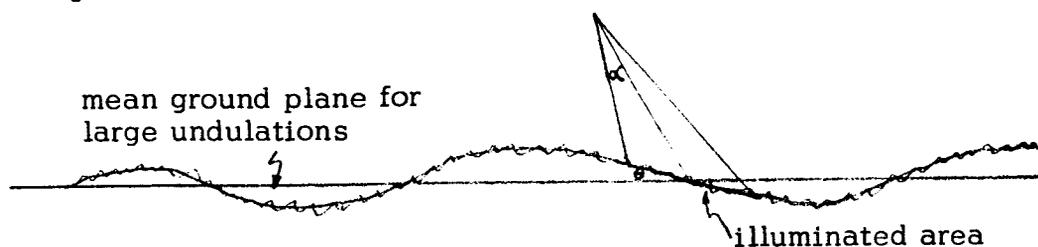


Figure IV-10

A different mean ground plane is needed if we want to calculate the average return power with respect to these big undulations. This can be done by writing (IV-21) as

$$P = K' \cos^2(\theta + \alpha) \int_0^{\infty} J_0[2k(\theta + \xi)] \xi \cdot \exp[-\{2k\sigma \cos(\theta + \alpha)\}^2 [1 - \rho(\xi)]] d\xi \quad (\text{IV-22})$$

and average over θ (see Figure IV-10). Taking $\rho(\xi)$ to be $\exp(-\xi/L)$, where L is the correlation distance and integrate we get

$$\begin{aligned} P &= K' (2k\sigma L)^2 \left[\{2k\sigma \cos(\theta + \alpha)\}^4 + \{2kL(\theta + \alpha)\}^2 \right]^{-3/2} \\ &= \frac{K' L^2}{(2k\sigma)^4 \cos^6(\theta + \alpha)} \left[1 + \left(\frac{L}{2k\sigma^2}\right)^2 \frac{(\theta + \alpha)^2}{\cos^4(\theta + \alpha)} \right]^{-3/2} \\ &\equiv K'' \cos^{-6}(\theta + \alpha) \left[1 + A \frac{(\theta + \alpha)^2}{\cos^4(\theta + \alpha)} \right]^{-3/2} \end{aligned} \quad (\text{IV-23})$$

where $K'' = \frac{K' L^2}{(2k\sigma)^4}$

$$A = \left(\frac{L}{2k\sigma^2}\right)^2$$

Assume $|\theta| < 3^\circ$ and $|A(\theta + \alpha)^2 / \cos^4(\theta + \alpha)| < 1$. Then, (IV-23) can be approximated as follows

$$P \approx \frac{K''}{\cos^6(\theta + \alpha)} \left\{ 1 - \frac{3}{2} A \left[\alpha^2 + 12\theta^2 \alpha^2 + 8\theta \alpha^3 + 8\theta^3 \alpha \right] \right\}$$

$$\begin{aligned}
& + 2\alpha^4 + 2\theta\alpha + \theta^2 + 2\theta^4] \\
& + \frac{3.5}{2.4} A^2 \left[6\theta^2\alpha^2 + 80\theta^3\alpha^3 + 60\theta^4\alpha^2 + 60\theta^2\alpha^4 \right. \\
& + 24\theta^5\alpha + 24\theta\alpha^5 + 4\theta^3\alpha + 4\theta\alpha^3 + \theta^4 \\
& \left. + \alpha^4 + 4\theta^6 + 4\alpha^6 \right] - \dots \quad \}
\end{aligned}$$

(IV-24)

If the large undulations were not present, the approximated power expression is

$$P \approx 1 - \frac{3}{2} A(\alpha^2 + 2\alpha^4) + \frac{3.5}{2.4} A^2(\alpha^4 + 4\alpha^6) - \dots \quad (\text{IV-25})$$

Comparison of (IV-24) with (IV-25) shows that the major effect, T , due to the presence of large undulations is, with higher order terms neglected,

$$T = -\frac{3}{2} A(\theta^2 + 2\theta\alpha) \quad (\text{IV-26})$$

If we assume that the probability distribution for θ is known, then the change in the mean power return due to these large undulations is

$$\bar{T} = -\frac{3A}{2} [m_2 + 2\alpha m_1] \quad (\text{IV-27})$$

where m_1 and m_2 are the first and second moments of θ respectively. It is thus seen that the large undulations will cause a drop in the mean return power.

The investigation in the previous chapters presents a complete theory based on Kirchhoff-Huygens principle on radar scattering together with some of the associated problems which help to clarify the physics of the radar return problem. A rather long expression, (III-24), was obtained for the mean scattered power in all directions which can, however, be simplified in various special cases. Comparison with experimental results then led to the investigation of the frequency dependence of the radar-measured statistical parameters of the surface and a more detailed autocorrelation function of the surface-heights,

$$\begin{aligned}
 P(\xi) + 1 + K^{-1} \ln \left\{ \frac{c}{a} \exp[-K(1 - e^{-|\xi|/L})] \right. \\
 + \frac{d}{a} \exp[-K(1 - e^{-|\xi|/l})] + \frac{f}{a} \exp[-K(1 - e^{-|\xi|/l'})] \\
 \left. + \frac{g}{a} \exp[-K(1 - e^{-|\xi|/l''})] \right\} ,
 \end{aligned}$$

(see Eq. IV-16).

The nature of frequency dependence of the said parameters was shown to be such that in general, only the effective parameters are obtained through fitting the experimental mean power return curves. The true parameters of the surface are measured only when the exploring wavelength is about four times the standard deviation of the surface and when the near-vertical incidence data are used. At larger angles of incidence the smaller structures are comparatively more effective and their character can be examined through the use of the more detailed autocorrelation function mentioned above. This novel correlation function was shown to give a more adequate description of the surface especially when more than one size of structures are present on the surface or when the distribution of structure sizes is not everywhere

continuous. For the case where all the structures are about the same size, this correlation function will reduce to a single exponential.

With this more detailed autocorrelation function and the knowledge of the nature of frequency dependence of the measured statistical parameters of the surface, estimates can then be obtained on these parameters. This is the first time estimates are obtained with frequency-dependent effect taken into account; this is also the first time smaller structures on the surface are distinguished and their sizes estimated. Other works cannot single out the effect of frequency dependence nor can they tell the presence of smaller structures. Thus, no meaningful estimate was possible, and those works can provide, at most, an explanation.

Since knowledge of the structure sizes on a given surface is the most important result that we have arrived at, we summarize below as to how estimates are obtained.

Consider a set of parameter values obtained through fitting experimental curves by the theory in Chapter IV. From Table II we have

Experimenter	Terrain	(cm)	A_0	A_1	A_2	l_0 (cm)	l_1 (cm)	l_2 (cm)
Pettengill	Moon	68	110	0.25	0.011	560	26.7	5.6
Taylor	Smooth Concrete	3	50	0.27		16.7	1.22	
Lynn et. al.	Moon	0.86	1.8	0.05	0.02	0.905	0.151	0.0955

where the l_i 's, $i = 0, 1, 2$, are the correlation distances and are obtained through the use of the argument on frequency dependence of these measured statistical parameters of the surface, namely, σ and l_i . The relations used are from Chapter IV,

$$\sigma \approx \lambda/4, \quad l_i \approx \frac{\sqrt{A_i} \pi \lambda}{4}$$

The parameters, σ , l_0 then characterize the largest structures seen at the wavelength, λ . Since σ is determined mainly by the large structures seen, the ratio σ/l_0 can be taken to be the rms slope in accordance with the definition of Evans and Pettengill [1963]. Presence of small structures is indicated by l_i , $i \geq 1$ but σ/l_i , $i \geq 1$ does not seem to have any meaning. This is easily seen in view of Figure V-1, where the largest structure drawn are the ones characterized by l_0 and the smaller structures are the ones characterized by any one of the l_i 's, $i \geq 1$.

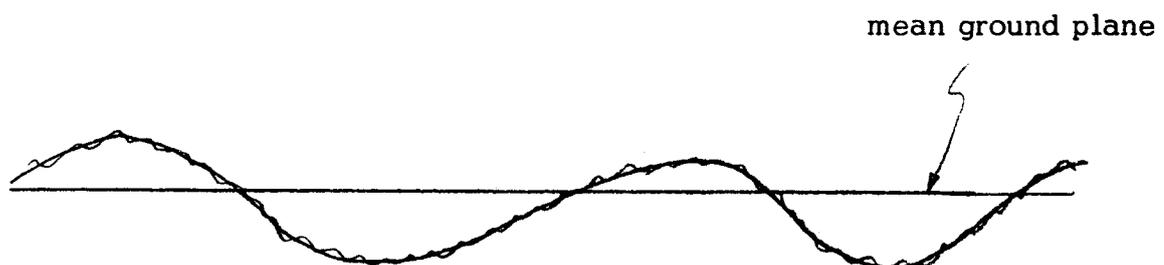


Figure V-1

A surface with two types of structures

It is worth emphasizing that the rms slope defined by σ/l_0 is not the actual rms slope of the surface, since the effect of smaller structures characterized by l_i , $i \geq 1$, have been ignored in its consideration and σ , l_0 are in general, frequency sensitive parameters. What is more, the rigorous definition of rms slope is

$\left\{ \overline{(\partial z / \partial \xi)^2} \right\}^{1/2}$ [Feinstein 1954] which does not necessarily coincide with σ/l_0 .

Since the main results are derived from the Kirchhoff method, they are obviously invalid where the method does not apply. Hence, future work on the same subject but for cases where the Kirchhoff method is not valid is of great importance. This can be done by either improving

the tangent plane approximation in the use of Huygens principle or employing a totally different approach. When necessary, the exact methods discussed in Chapter II may be used. However, many curves have to be plotted using a high speed computer before any physical insight can be gained.

A preliminary study on the effects of the size of the illuminated area on radar measurements at near-vertical incidence comes out with the result that the presence of large undulations comparable in size to the illuminated area will cause a drop in the mean return power.

APPENDIX 1

PROOF OF KARHUNEN-LOÉVE THEOREM

Theorem: A random process defined by the sample function $Z(x, y)$ continuous in the mean on a closed set, D , has on D an orthogonal decomposition,

$$Z(x, y) = \sum_{n,m} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(x, y) z_{mn}, \quad (x, y) \in D$$

with

$$\int_D \varphi_{mn}(x, y) \varphi_{pq}(x, y) dx dy = \delta_{mp} \delta_{nq}$$

$$\overline{z_{mn} z_{pq}} = \delta_{mp} \delta_{nq}$$

if and only if the λ_{mn} are the eigen values and the $\varphi_{mn}(x, y)$ are the orthonormalized eigen functions of its correlation function. Then, the series converges in the mean on D uniformly. (We give a proof below for a real random process).

Proof: \Rightarrow Show λ_{mn} , $\varphi_{mn}(x, y)$ are the eigen values and eigen functions of the correlation function.

$$r(x, y; x', y') \equiv \overline{Z(x, y) Z(x', y')}$$

$$= \sum_{p,q} \sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \lambda_{pq}^{-\frac{1}{2}} \varphi_{mn}(x, y) \varphi_{pq}(x', y') \overline{z_{mn} z_{pq}}$$

$$= \sum_{p,q} \sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \lambda_{pq}^{-\frac{1}{2}} \varphi_{mn}(x, y) \varphi_{pq}(x', y') \delta_{mp} \delta_{nq}$$

$$= \sum_{m,n} \lambda_{mn}^{-1} \varphi_{mn}(x, y) \varphi_{mn}(x', y')$$

$$\begin{aligned}
& \int_D \rho(x, y; x', y') \varphi_{pq}(x', y') dx' dy' \\
&= \sum_{m,n} \lambda_{mn}^{-1} \varphi_{mn}(x, y) \int_D \varphi_{mn}(x', y') \varphi_{pq}(x', y') dx' dy' \\
&= \lambda_{pq}^{-1} \varphi_{pq}(x, y)
\end{aligned}$$

This shows that λ_{pq} 's are the eigen values and the $\varphi_{pq}(x, y)$'s are the orthonormalized eigen functions of its correlation function, $\rho(x, y; x', y')$.

⊕ Using the fact that $\varphi_{mn}(x, y)$ are orthonormalized eigen functions of $\rho(x, y; x', y')$, we shall show that the random coefficients are orthogonal in the sense defined. Also, $\sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(x, y) Z_{mn}$ converges to $Z(x, y)$ in the mean.

Let us define the random coefficients Z_{pq} by

$$Z_{pq} \lambda_{pq}^{-\frac{1}{2}} = \int_D Z(x, y) \varphi_{pq}(x, y) dx dy$$

Then,

$$\lambda_{mn}^{-\frac{1}{2}} \lambda_{pq}^{-\frac{1}{2}} Z_{mn} Z_{pq} = \int_D Z(x', y') \varphi_{mn}(x', y') dx' dy' \int_D Z(x, y) \varphi_{pq}(x, y) dx dy$$

$$\lambda_{mn}^{-\frac{1}{2}} \lambda_{pq}^{-\frac{1}{2}} \overline{Z_{mn} Z_{pq}} = \iint_{D \times D} \rho(x, y; x', y') \varphi_{pq}(x, y) \varphi_{mn}(x', y') dx dx' dy dy'$$

Since $\varphi_{mn}(x, y) = \lambda_{mn} \int_D \rho(x, y; x', y') \varphi_{mn}(x', y') dx' dy'$, we have

$$\lambda_{mn}^{-\frac{1}{2}} \lambda_{pq}^{-\frac{1}{2}} \overline{Z_{pq} Z_{mn}} = \int_D \lambda_{mn}^{-1} \varphi_{mn}(x, y) \varphi_{pq}(x, y) dx dy = \lambda_{mn}^{-1} \delta_{mp} \delta_{nq}$$

$$\therefore \overline{Z_{pq} Z_{mn}} = \delta_{mp} \delta_{nq}$$

since $\lambda_{mn} = 0$ is not an admissible eigen value.

To show that $\sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(x,y) Z_{mn}$ converges to $Z(x,y)$ in the mean, let

$$Z_{NM}(x,y) = \sum_{m,n}^{N,M} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(x,y) Z_{mn}$$

A direct calculation shows that

$$\begin{aligned} \overline{Z(x,y) Z_{NM}(x,y)} &= \sum_{m,n}^{M,N} \sum_{p,q} \lambda_{mn}^{-\frac{1}{2}} \lambda_{pq}^{-\frac{1}{2}} \varphi_{mn}(x,y) \varphi_{pq}(x,y) \overline{Z_{mn} Z_{pq}} \\ &= \sum_{m,n}^{M,N} \lambda_{mn}^{-1} \varphi_{mn}^2(x,y). \end{aligned}$$

Hence,

$$\begin{aligned} \overline{[Z(x,y) - Z_{NM}(x,y)]^2} &= \overline{Z(x,y)Z(x,y) - 2Z(x,y)Z_{NM}(x,y)} \\ &\quad + \overline{Z_{NM}(x,y)Z_{NM}(x,y)} \\ &= \rho(x,y;x,y) - 2 \sum_{m,n}^{M,N} \lambda_{mn}^{-1} \varphi_{mn}^2(x,y) - \sum_{m,n}^{M,N} \lambda_{mn}^{-1} \varphi_{mn}^2(x,y) \\ &= \rho(x,y;x,y) - \sum_{m,n}^{M,N} \lambda_{mn}^{-1} \varphi_{mn}^2(x,y) \end{aligned}$$

Now Mercer's Theorem [Courant and Hilbert I p. 138] states that

$$\lim_{\substack{N \rightarrow \infty \\ M \rightarrow \infty}} \sum_{m,n}^{N,M} \lambda_{mn}^{-1} \varphi_{mn}^2(x,y) = \rho(x,y;x,y)$$

Thus,

$$\lim_{\substack{N \rightarrow \infty \\ M \rightarrow \infty}} \frac{[Z(\alpha, y) - Z_{NM}(\alpha, y)]^2}{[Z(\alpha, y) - Z_{NM}(\alpha, y)]^2} = \lim_{\substack{N \rightarrow \infty \\ M \rightarrow \infty}} [P(\alpha, y; \alpha, y) - \sum_{m,n}^{N,M} \lambda_{mn}^{-1} \varphi_{mn}^2(\alpha, y)]$$

$$= 0$$

Hence, by definition

$$Z(\alpha, y) = \text{l.i.m.}_{\substack{N \rightarrow \infty \\ M \rightarrow \infty}} \sum_{m,n}^{N,M} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(\alpha, y) Z_{mn}$$

APPENDIX 2
ENSEMBLE AVERAGES

The ensemble averages of the following quantities are found below

$$\begin{aligned}
 \text{(i)} \quad \overline{e^{ikBz}} &= \overline{\exp\left[ikB \sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn} z_{mn} \right]} \\
 &= \int_{-\infty}^{\infty} e^{ikBz} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{z^2}{2\sigma^2}} dz \\
 &= \frac{e^{-\frac{(\sigma B k)^2}{2}}}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{z}{\sigma} - i\sigma B k\right)^2} dz \\
 &= e^{-\frac{(\sigma B k)^2}{2}} \frac{2\sqrt{\pi}\sigma}{\sqrt{2\pi\sigma^2}} \\
 &= \exp\left[-\frac{\sigma^2 B^2 k^2}{2} \right]
 \end{aligned}
 \tag{2-1}$$

Since

$$\begin{aligned}
 &\prod_{m,n} \left[e^{ikB \lambda_{m,n}^{-\frac{1}{2}} \varphi_{mn}(x,y) z_{mn}} \right] \\
 &= \prod_{m,n} \left[\exp\left(-\frac{k^2 B^2 \lambda_{mn}^{-1} \varphi_{mn}^2(x,y) \sigma_{mn}^2}{2} \right) \right] \\
 &= \exp\left[-\frac{k^2 B^2 \sum \lambda_{mn}^{-1} \varphi_{mn}^2(x,y)}{2} \right]
 \end{aligned}
 \tag{2-2}$$

Hence (2-1) and (2-2) implies $\sigma^2 = \sum_{m,n} \lambda_{m,n}^{-1} \varphi_{mn}^2(x,y)$
 (Recall, $\sigma_{mn}^2 = 1$)

(ii)

$$\begin{aligned}
 \overline{Z_x e^{ikBz}} &= \sum_{mn} \lambda_{mn}^{-\frac{1}{2}} \varphi_{xmn}(x,y) z_{mn} e^{ikB \sum \lambda_{pf}^{-\frac{1}{2}} \varphi_{pf}(x,y) z_{pf}} \\
 &= \sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{xmn}(x,y) \left[z_{mn} e^{ikB \sum \lambda_{pf}^{-\frac{1}{2}} \varphi_{pf}(x,y) z_{pf}} \right] \\
 &= \sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{xmn}(x,y) \left\{ -i \frac{d}{dB_{mn}} \left(\prod_{pf} e^{iB_{pf} z_{pf}} \right) \right\} \\
 &= \sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{xmn}(x,y) \left\{ i B_{mn} \prod_{pf} e^{-\frac{1}{2} B_{pf}^2 z_{pf}^2} \right\} \\
 &= i \sum_{mn} \lambda_{mn}^{-\frac{1}{2}} \varphi_{xmn}(x,y) \left[k B \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(x,y) e^{-\frac{1}{2} \sum B_{pf}^2 z_{pf}^2} \right] \\
 &= i \frac{1}{2} \frac{\partial}{\partial x} \sigma^2 \exp \left[-\frac{1}{2} k^2 B^2 \sum_{pf} \lambda_{pf}^{-1} \varphi_{pf}^2(x,y) \right] \\
 &= i \frac{1}{2} \frac{\partial}{\partial x} \sigma^2 \exp \left[-\frac{1}{2} k^2 B^2 \sigma^2 \right]
 \end{aligned}$$

(2-3)

where $B_{pf} = k B \lambda_{pf}^{-\frac{1}{2}} \varphi_{pf}(x,y)$

$$\begin{aligned}
& \text{(iii)} \quad e^{i k B (z - z')} \\
& = \overline{\exp \left[i k B \left(\sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(x, y) Z_{mn} - \sum_{m,n} \lambda_{mn}^{-\frac{1}{2}} \varphi_{mn}(x', y') Z_{mn} \right) \right]} \\
& = \overline{\exp \left[i k B \sum_{m,n} \left\{ \lambda_{mn}^{-\frac{1}{2}} \left[\varphi_{mn}(x, y) - \varphi_{mn}(x', y') \right] Z_{mn} \right\} \right]} \\
& = \overline{\exp \left[i \sum_{m,n} C_{mn} Z_{mn} \right]} \\
& = \exp \left[-\frac{1}{2} \sum_{m,n} C_{mn}^2 \right] \\
& = \exp \left\{ -\frac{1}{2} k^2 B^2 \sum_{m,n} \lambda_{mn}^{-1} \left[\varphi_{mn}^2(x, y) + \varphi_{mn}^2(x', y') - 2 \varphi_{mn}(x, y) \varphi_{mn}(x', y') \right] \right\} \\
& = \exp \left\{ -\frac{k^2 B^2}{2} \left(\sigma^2 + \sigma'^2 - 2 \rho(x, y; x', y') \right) \right\}
\end{aligned}$$

where

$$\begin{aligned}
C_{mn} &= k B \lambda_{mn}^{-\frac{1}{2}} \left\{ \varphi_{mn}(x, y) - \varphi_{mn}(x', y') \right\} \\
\sigma &= \sigma(x, y) \quad , \quad \sigma' = \sigma(x', y')
\end{aligned} \tag{2-4}$$

For stationary random process, σ is a constant and (2-4) becomes

$$\exp[i k B (z - z')] = \exp[-k^2 B^2 (\sigma^2 - \rho(x, y; x', y'))] \tag{2-5}$$

$$(iv) \quad \overline{Z_x e^{ik_B(z-z')}}} \\ = \sum_{mn} \lambda_{mn}^{-1} \varphi_{xmn}(x,y) \overline{[Z_{mn} e^{ik_B(z-z')}]}$$

$$\text{where } \overline{[Z_{mn} e^{ik_B(z-z')}]} \\ = \overline{Z_{mn} \exp\{ik_B \sum_{p,q} \lambda_{pq}^{-\frac{1}{2}} [\varphi_{pq}(x,y) - \varphi_{pq}(x',y')] Z_{pq}\}} \\ = \overline{Z_{mn} \exp\{i \sum_{p,q} C_{pq} Z_{pq}\}} \\ = \left\{ -i \frac{k}{\lambda C_{mn}} \prod_{p,q} \exp[i C_{pq} Z_{pq}] \right\} \\ = i C_{mn} \exp\left[-\frac{1}{2} \sum_{p,q} C_{pq}^2\right]$$

$$\therefore \overline{Z_x e^{ik_B(z-z')}} \\ = ik_B \sum_{m,n} \lambda_{mn}^{-1} [\varphi_{xmn}(x,y) \varphi_{mn}(x,y) - \varphi_{xmn}(x,y) \varphi_{mn}(x',y')] \\ \exp\left[-\frac{1}{2} \sum_{p,q} C_{pq}^2\right] \\ = ik_B \left[\frac{1}{2} \frac{\partial}{\partial x} \sigma^2 - \frac{\partial}{\partial x} \rho(x,y; x',y') \right] \\ \cdot \exp\left[-\frac{k^2 B^2}{2} \sum_{m,n} \lambda_{mn}^{-1} \{\varphi_{mn}^2(x,y) + \varphi_{mn}^2(x',y') - 2\varphi_{mn}(x,y)\varphi_{mn}(x',y')\}\right] \\ = ik_B \left[\frac{1}{2} \frac{\partial}{\partial x} \sigma^2 - \frac{\partial \rho}{\partial x} \right] \exp\left[-\frac{k^2 B^2}{2} \{\sigma^2 + \sigma'^2 - 2\rho\}\right]$$

(2-6)

For stationary random process,

$$\overline{Z_x e^{ikB(z-z')}} = ikB \frac{\partial \rho}{\partial u} \exp[-k^2 B^2 (\sigma^2 - \rho)] \quad (2-7)$$

where $u = x' - x$, $\rho = \rho(x, y; x', y')$

Similarly,

$$\begin{aligned} \overline{Z_{x'} e^{ikB(z-z')}} &= ikB \left[\frac{\partial \rho}{\partial x'} - \frac{1}{2} \frac{\partial}{\partial x'} \sigma'^2 \right] \\ &\cdot \exp \left\{ -\frac{k^2 B^2}{2} [\sigma^2 + \sigma'^2 - 2\rho] \right\} \end{aligned} \quad (2-8)$$

For stationary process, (2-8) becomes

$$\overline{Z_{x'} e^{ikB(z-z')}} = ikB \frac{\partial \rho}{\partial u} \exp[-k^2 B^2 (\sigma^2 - \rho)] \quad (2-9)$$

(v)

$$\begin{aligned} \overline{Z_x Z_{x'} e^{ikB(z-z')}} &= \sum_{m,n} \sum_{p,q} \lambda_{mn}^{-\frac{1}{2}} \varphi_{xmn}(x,y) \lambda_{pq}^{-\frac{1}{2}} \varphi_{x'pq}(x',y') \overline{Z_{mn} Z_{pq} e^{i \sum C_{st} Z_{st}}} \end{aligned}$$

where $\overline{Z_{mn} Z_{pq} e^{i \sum C_{st} Z_{st}}}$

$$= - \frac{d}{dC_{mn}} \frac{d}{dC_{pq}} \overline{\left[\prod_{st} e^{i C_{st} Z_{st}} \right]}$$

$$= - \frac{d}{dC_{mn}} \frac{d}{dC_{pq}} \left\{ e^{-\frac{1}{2} \sum_{st} C_{st}^2} \right\}$$

$$= \begin{cases} (1 - C_{mn}^2) \exp\left[-\frac{1}{2} \sum_{st} C_{st}^2\right], & \text{if } m = p, n = q \\ -C_{mn} C_{pq} \exp\left[-\frac{1}{2} \sum_{st} C_{st}^2\right], & \text{if } m \neq p, n \neq q \\ & \text{or both.} \end{cases}$$

$$\therefore \overline{Z_x Z_{x'}} e^{ikB(z-z')}$$

$$\begin{aligned} &= \sum_{m,n} \lambda_{mn}^{-1} \varphi_{xmn} \varphi'_{xmn} \left[1 - \frac{k^2 B^2}{\lambda_{mn}} (\varphi_{mn}^2 + \varphi'_{mn}{}^2 - 2\varphi_{mn} \varphi'_{mn}) \right] \\ &\quad \cdot e^{-\frac{k^2 B^2}{2} (\sigma^2 + \sigma'^2 - 2\rho)} - \left\{ \sum_{mn} \sum_{pq} \lambda_{mn}^{-\frac{1}{2}} \varphi_{xmn} \lambda_{pq}^{-\frac{1}{2}} \varphi_{xpq}(x', y') \right. \\ &\quad \left. k^2 B^2 \lambda_{mn}^{-\frac{1}{2}} \lambda_{pq}^{-\frac{1}{2}} [\varphi_{mn} \varphi_{pq} + \varphi'_{mn} \varphi'_{pq} - \varphi_{mn} \varphi'_{pq} - \varphi'_{mn} \varphi_{pq}] \right\} \\ &\quad \cdot \exp\left[-\frac{k^2 B^2}{2} (\sigma^2 + \sigma'^2 - 2\rho)\right] \\ &= \left\{ \frac{\partial^2 \rho}{\partial x \partial x'} - k^2 B^2 \left[\frac{1}{2} \frac{\partial \sigma^2}{\partial x} \frac{\partial \rho}{\partial x'} + \frac{1}{2} \frac{\partial \rho}{\partial x} \frac{\partial \sigma'^2}{\partial x'} - \frac{1}{4} \frac{\partial \sigma^2}{\partial x} \frac{\partial \sigma'^2}{\partial x'} \right. \right. \\ &\quad \left. \left. - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial x'} \right] \right\} \exp\left[-\frac{k^2 B^2}{2} (\sigma^2 + \sigma'^2 - 2\rho)\right] \quad (2-10) \end{aligned}$$

For stationary process

$$Z_x Z_{x'} e^{ikB(z-z')} = - \left\{ \frac{\partial^2 \rho}{\partial u^2} + k^2 B^2 \left(\frac{\partial \rho}{\partial u} \right)^2 \right\} e^{-k^2 B^2 (\sigma^2 - \rho)} \quad (2-11)$$

APPENDIX 3
TRANSFORMATION OF COORDINATES

Since we want to integrate over the illuminated area, it is convenient if we express our x, y coordinate in terms of some appropriate coordinate S, γ [see Figure A-1]

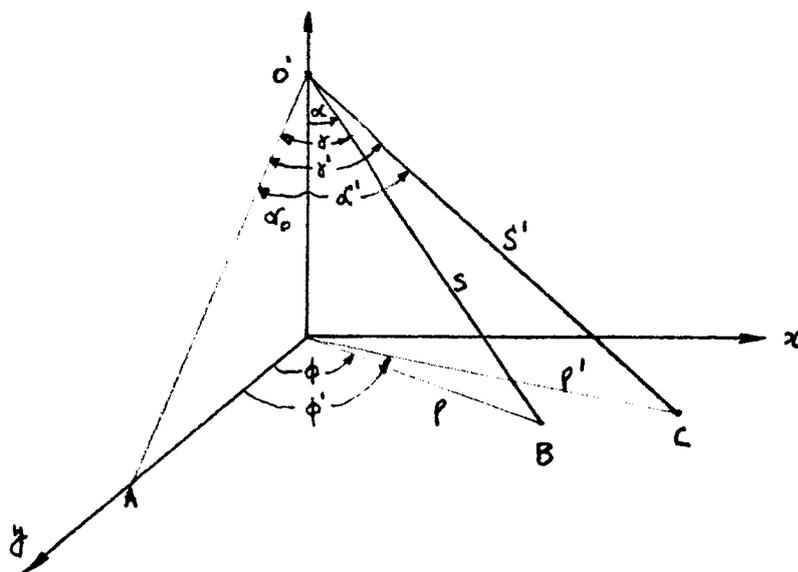


Figure A-1
Geometry of the radar problem

We can accomplish this by going in two steps: (i) express x, y in terms of ρ and ϕ and (ii) express ρ and ϕ in terms of S and γ . Thus, from Figure A-1,

$$x = \rho \sin \phi \quad (\text{A-3-1})$$

$$y = \rho \cos \phi \quad (\text{A-3-2})$$

$$p = S \sin \alpha$$

$$\phi = \cos^{-1} \left[\frac{\cos \delta - A \cos \alpha}{B \sin \alpha} \right]$$

where $A = \cos \alpha_0$

$B = \sin \alpha_0$

The fact that $\cos \phi = \frac{\cos \delta - A \cos \alpha}{B \sin \alpha}$ will be

shown later. Now (A-3-1) and (A-3-2) can be written

$$x = S \sin \alpha \sin \phi = S \sin \alpha \sin \left\{ \cos^{-1} \left[\frac{\cos \delta - A \cos \alpha}{B \sin \alpha} \right] \right\}$$

$$y = S \sin \alpha \cos \phi = \frac{S}{B} [\cos \delta - A \cos \alpha] \quad (\text{A-3-3})$$

$$\frac{\partial y}{\partial S} = \sin \alpha \cos \phi + S \cos \phi \cos \alpha \frac{d\alpha}{dS}$$

$$= \sin \alpha \cos \phi + \cos \phi \cos \alpha \cot \alpha$$

$$= \cos \phi \csc \alpha$$

$$\frac{\partial y}{\partial S} = S \sin \alpha \frac{d}{d\delta} \left[\frac{\cos \delta - A \cos \alpha}{B \sin \alpha} \right]$$

$$= -\frac{S}{B} \sin \delta$$

$$\frac{\partial x}{\partial S} = \sin \alpha \sin \phi + S \sin \phi \cos \alpha \frac{d\alpha}{dS}$$

$$= \sin \alpha \sin \phi + \sin \phi \cos \alpha \cot \alpha$$

$$= \sin \phi \csc \alpha$$

$$\begin{aligned}\frac{\partial z}{\partial r} &= S \sin \alpha \cos \phi \frac{\sin \tau}{B \sin \alpha \sin \phi} \\ &= (S/B) \sin \tau \cot \phi\end{aligned}$$

Thus, the Jacobian is

$$J = \begin{vmatrix} \cos \phi \csc \alpha & -\frac{S}{B} \sin \tau \\ \sin \phi \csc \alpha & \frac{S}{B} \cot \phi \sin \tau \end{vmatrix}$$

$$= \frac{S}{B} \left(\frac{\cos^2 \phi}{\sin \phi} \csc \alpha \sin \tau + \sin \phi \csc \alpha \sin \tau \right)$$

$$= \frac{S}{B} \csc \alpha \sin \tau \csc \phi$$

(A-3-4)

$$\text{The element of area} = \left| \frac{S}{B} \csc \alpha \sin \tau \csc \phi \right| dS d\tau$$

$$\text{Let } s' = s + \tau$$

$$r' = r + \varphi$$

$$\alpha' = \alpha + \eta$$

$$y' - y = (s+t) \sin(\alpha+\eta) \left[\frac{\cos(\delta+\varphi) - A \cos(\alpha+\eta)}{B \sin(\alpha+\eta)} \right]$$

$$- S \sin \alpha \left[\frac{\cos \delta - A \cos \alpha}{B \sin \alpha} \right]$$

$$= \frac{1}{B} \left\{ (s+t) [\cos(\delta+\varphi) - A \cos(\alpha+\eta)] - S (\cos \delta - A \cos \alpha) \right\}$$

$$= \frac{1}{B} \left\{ (s+t) [\cos \delta \cos \varphi - \sin \delta \sin \varphi - A \cos \alpha \cos \eta + A \sin \alpha \sin \eta] - S (\cos \delta - A \cos \alpha) \right\}$$

$$\approx \frac{1}{B} \left\{ t \cos \delta \cos \varphi - S \sin \delta \sin \varphi - t A \cos \alpha \cos \eta + S A \sin \alpha \sin \eta \right\}$$

$$= \frac{1}{B} \left\{ t (\cos \delta \cos \varphi - A \cos \alpha \cos \eta) + S (A \sin \alpha \sin \eta - \sin \delta \sin \varphi) \right\}$$

$$\approx \frac{1}{B} \left\{ t (\cos \delta - A \cos \alpha) + S (A \eta \sin \alpha - \varphi \sin \delta) \right\}$$

$$x' - x = (s+t) \sin(\alpha+\eta) \left[1 - \left\{ \frac{\cos(\theta+\varphi) - A \cos(\alpha+\eta)}{B \sin(\alpha+\eta)} \right\}^2 \right]^{\frac{1}{2}}$$

$$= s \sin \alpha \left[1 - \left(\frac{\cos \theta - A \cos \alpha}{B \sin \alpha} \right)^2 \right]^{\frac{1}{2}}$$

$$\approx (s+t) \sin(\alpha+\eta) \left\{ 1 - \left[\cos^2(\theta+\varphi) - 2A \cos(\theta+\varphi) \cos(\alpha+\eta) + A^2 \cos^2(\alpha+\eta) \right] / 2B^2 \sin^2(\alpha+\eta) \right\}$$

$$= s \sin \alpha \left[1 - \frac{\cos^2 \theta - 2A \cos \theta \cos \alpha + A^2 \cos^2 \alpha}{2B^2 \sin^2 \alpha} \right]$$

$$\approx (s+t) \sin(\alpha+\eta) - \frac{(s+t) \csc(\alpha+\eta)}{2B^2} \left[\cos^2 \theta \cos^2 \varphi \right.$$

$$\left. - 2 \cos \theta \cos \varphi \sin \theta \sin \varphi - 2A (\cos \theta \cos \varphi \cos \alpha \cos \eta) \right.$$

$$\left. - \cos \theta \cos \varphi \sin \alpha \sin \eta - \sin \theta \sin \varphi \cos \alpha \cos \eta) \right.$$

$$\left. + A^2 (\cos^2 \alpha \cos^2 \eta - 2 \cos \alpha \cos \eta \sin \alpha \sin \eta) \right]$$

$$= s \sin \alpha + \frac{s}{2B^2} \csc \alpha \left[\cos^2 \theta - 2A \cos \theta \cos \alpha + A^2 \cos^2 \alpha \right]$$

$$\approx t \sin \alpha \cos \eta + s \cos \alpha \sin \eta - \frac{t}{2B^2} \csc \alpha \cos^2 \theta \cos^2 \varphi$$

$$+ \frac{s}{B^2} \csc \alpha \cos \theta \cos \varphi \sin \theta \sin \varphi + \frac{At}{B^2} \csc \alpha \cos \theta \cos \varphi \cos \alpha \cos \eta$$

$$\begin{aligned}
& -\frac{SA}{B^2} \csc \alpha \cos \delta \cos \varphi \sin \alpha \sin \eta - \frac{SA}{B^2} \csc \alpha \sin \delta \sin \varphi \cos \alpha \cos \eta \\
& -\frac{tA^2}{2B^2} \csc \alpha \cos^2 \alpha \cos^2 \eta + \frac{SA^2}{B^2} \csc \alpha \cos \alpha \cos \eta \sin \alpha \sin \eta \\
= & t \left[\sin \alpha \cos \eta - \frac{1}{2B^2} \csc \alpha \cos^2 \delta \cos^2 \varphi \right. \\
& \left. + \frac{A}{B^2} \csc \alpha \cos \delta \cos \varphi \cos \alpha \cos \eta - \frac{A^2}{2B^2} \csc \alpha \cos^2 \alpha \cos^2 \eta \right] \\
& + \frac{S}{B^2} \csc \alpha \left[\cos \delta \cos \varphi \sin \delta \sin \varphi - A \cos \delta \cos \varphi \sin \alpha \sin \eta \right. \\
& \left. - A \sin \delta \sin \varphi \cos \alpha \cos \eta + A^2 \cos \alpha \cos \eta \sin \alpha \sin \eta \right] \\
& + S \cos \alpha \sin \eta \\
= & t \sin \alpha \cos \eta + \frac{t}{B^2} \csc \alpha \left[-\frac{1}{2} \cos^2 \delta \cos^2 \varphi \right. \\
& \left. + A \cos \delta \cos \varphi \cos \alpha \cos \eta - \frac{A^2}{2} \cos^2 \alpha \cos^2 \eta \right] \\
& + S \cos \alpha \sin \eta + \frac{S}{B^2} \csc \alpha \left[\cos \delta \cos \varphi \sin \delta \sin \varphi \right. \\
& \left. - A \cos \delta \cos \varphi \sin \alpha \sin \eta - A \sin \delta \sin \varphi \cos \alpha \cos \eta \right. \\
& \left. + A^2 \cos \alpha \cos \eta \sin \alpha \sin \eta \right]
\end{aligned}$$

(A-3-6)

If we consider the case of narrow beam and short pulse problem with sufficiently large angle of incidence, then we can choose our reference axes to be such that

$$\begin{aligned}\alpha_0 &\approx \pi/2 \\ \phi &\approx \pi/2\end{aligned}\quad (\text{A-3-7})$$

This choice implies $\gamma \approx \pi/2$. Hence the Equations, (A-3-4), (A-3-5) and (A-3-6) becomes

$$J = S \csc \alpha \frac{1}{B} \sin \tau \csc \phi \approx S \csc \alpha \quad (\text{A-3-8})$$

$$\begin{aligned}y' - y &= \frac{1}{B} \{t (\cos \tau - A \cos \alpha) + S (A \eta \sin \alpha - \varphi \sin \tau)\} \\ &\approx -S \varphi\end{aligned}\quad (\text{A-3-9})$$

$$\begin{aligned}x' - x &= t \sin \alpha \cos \eta + \frac{t}{B^2} \csc \alpha \left[-\frac{1}{2} \cos^2 \tau \cos^2 \varphi \right. \\ &\quad \left. + A \cos \tau \cos \varphi \cos \alpha \cos \eta - \frac{A^2}{2} \cos^2 \alpha \cos^2 \eta \right] \\ &\quad + S \cos \alpha \sin \eta + \frac{S}{B^2} \csc \alpha \left[\cos \tau \cos \varphi \sin \tau \sin \varphi \right. \\ &\quad \left. - A \cos \tau \cos \varphi \sin \alpha \sin \eta - A \sin \tau \sin \varphi \cos \alpha \cos \eta \right. \\ &\quad \left. + A^2 \cos \alpha \cos \eta \sin \alpha \sin \eta \right] \\ &\approx t \sin \alpha + S \eta \cos \alpha \\ &= t \sin \alpha + t \cot \alpha \cos \alpha = t \operatorname{csc} \alpha\end{aligned}$$

Note that the choice of reference leads to

$$A = 0,$$

$$B = 1.$$

To find ϕ in terms of S and γ .

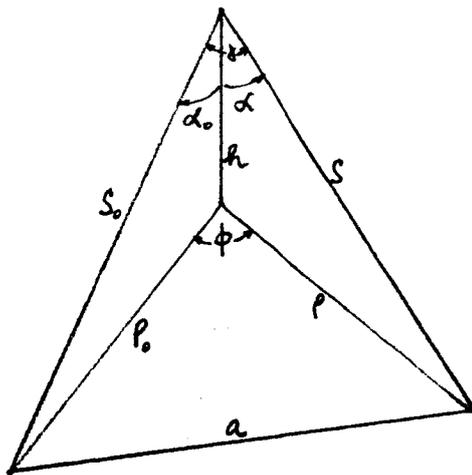


Figure A-2

Geometry for relating ϕ to S and γ .

From Figure A-2, we have

$$\begin{aligned} a^2 &= p^2 + p_0^2 - 2pp_0 \cos \phi \\ &= (S \sin \alpha)^2 + (S_0 \sin \alpha_0)^2 - 2SS_0 \sin \alpha \sin \alpha_0 \cos \phi \end{aligned} \quad (\text{A-3-11})$$

$$\text{Also, } a^2 = S^2 + S_0^2 - 2SS_0 \cos \gamma \quad (\text{A-3-12})$$

Noting that $S = \frac{h}{\cos \alpha}$, $S_0 = \frac{h}{\cos \alpha_0}$, we have

from (A-3-11) and (A-3-12),

$$\begin{aligned}
& \frac{1}{\cos^2 \alpha} + \frac{1}{\cos^2 \alpha_0} - \frac{2 \cos \tau}{\cos \alpha \cos \alpha_0} \\
&= \tan^2 \alpha + \tan^2 \alpha_0 - 2 \tan \alpha \tan \alpha_0 \cos \phi \\
& \cos^2 \alpha \cos^2 \alpha_0 - 2 \cos \alpha \cos \alpha_0 \cos \tau \\
&= \sin^2 \alpha \cos^2 \alpha_0 + \sin^2 \alpha_0 \cos^2 \alpha - 2 \sin \alpha \sin \alpha_0 \cos \alpha \cos \alpha_0 \cos \phi \\
& 2 \cos^2 \alpha \cos^2 \alpha_0 = 2 \cos \alpha \cos \alpha_0 (\cos \tau - \sin \alpha \sin \alpha_0 \cos \phi) \\
& \cos \alpha \cos \alpha_0 = \cos \tau - \sin \alpha \sin \alpha_0 \cos \phi \\
& \therefore \cos \phi = \frac{\cos \tau - \cos \alpha \cos \alpha_0}{\sin \alpha \sin \alpha_0}
\end{aligned}$$

(A-3-13)

APPENDIX 4

EVALUATION OF INTEGRALS IN (III-23)

(I) Consider
$$K_1 \int A_0 \exp[-i\gamma\xi - K(1 - e^{-\xi/a})] \xi d\xi d\theta' d\phi$$

$S_0 \cos\alpha \xi d\xi d\theta' d\phi$

(1)

where

$$A_0 = 1 - \sin^2\theta \sin^2\phi$$

$$\gamma = -p \sin\theta' + q \cos\theta'$$

$$p = k(\cos\alpha - \sin\theta \sin\phi)$$

$$q = k \sin\theta \cos\phi$$

$$K = k^2 \sigma^2 B^2$$

$$K_1 = \frac{1}{2} \eta H_0^2 / R^2 \lambda^2$$

This integral can be written as

$$K_1 \iiint A_0 e^{-K - i\gamma\xi + K e^{-\xi/a}} \xi d\xi d\theta' S_0 \cos\alpha d\phi$$

$$= K_1 A_0 e^{-K} \sum_{n=0}^{\infty} \frac{K^n}{n!} \iiint e^{-i\gamma\xi - \frac{n\xi}{a}} \xi d\xi d\theta' S_0 \cos\alpha d\phi$$

(2)

Now consider the following integral

$$\int_0^{2\pi} \int_0^{\infty} e^{-(\frac{n}{a} + ip \sin\theta' - iq \cos\theta')\xi} \xi d\xi d\theta' \quad (n > 0)$$

$$= \int_0^{2\pi} \frac{d\theta'}{(\frac{n}{a} + ip \sin\theta' - iq \cos\theta')^2}$$

(3)

Let $\cos \theta' = (z + z^{-1})/2$, $\sin \theta' = (z - z^{-1})/2i$ and $d\theta' = dz/iz$.

Then (3) becomes

$$\begin{aligned} & \oint \frac{dz}{\left[\frac{n}{a} + \frac{p}{2}(z - z^{-1}) - i\frac{q}{2}(z + z^{-1}) \right]^2 iz} \\ &= \oint \frac{-iz dz}{\left[\left(\frac{p}{2} - i\frac{q}{2} \right) z^2 + \frac{n}{a} z - \left(\frac{p}{2} + i\frac{q}{2} \right) \right]^2} \\ &= \oint \frac{-4iz dz}{\left[(z - z_1)^2 (z - z_2)^2 \right] (p - iq)^2} \end{aligned}$$

where $\left. \begin{array}{l} z_1 \text{ (plus sign)} \\ z_2 \text{ (minus sign)} \end{array} \right\} = \frac{-\frac{n}{a} \pm \sqrt{\left(\frac{n}{a}\right)^2 + p^2 + q^2}}{p - iq}$

Residue due to z_1 aside from the factor $4/(p - iq)^2$

$$\begin{aligned} &= \frac{d}{dz} \left[\frac{-iz}{(z - z_2)^2} \right]_{z=z_1} = \frac{-i(z - z_2)^2 + iz_2 2(z - z_2)}{(z - z_2)^4} \Big|_{z=z_1} \\ &= \frac{i(z + z_2)}{(z - z_2)^3} \Big|_{z=z_1} = \frac{in(p - iq)^2}{4a \left[\left(\frac{n}{a}\right)^2 + p^2 + q^2 \right]^{3/2}} \end{aligned}$$

Thus (3) becomes

$$\begin{aligned} 2\pi i \text{ Residue} &= \frac{2\pi n}{a} \frac{1}{\left[\left(\frac{n}{a}\right)^2 + p^2 + q^2 \right]^{3/2}} \\ &= 2\pi n a^2 \left[n^2 + a^2(p^2 + q^2) \right]^{-3/2} \quad (4) \end{aligned}$$

For the case, $n = 0$, the limits for the variables τ , and φ and consequently ξ will give rise to a delta function behavior. However, these limits are approximate values. Actually, integration should be carried out over a finite area. Thus, a more accurate result is to go back to (22) and for an illuminated area of size $2l \times 2l$ we have instead of (3)

$$\begin{aligned} & \int_{-l}^l \int_{-l}^l \exp[-ipt \csc \alpha + if \delta \varphi] d(\tau \csc \alpha) d(\delta \varphi) \\ &= \frac{e^{-ipl} - e^{ipl}}{(-ip)(if)} (e^{ifl} - e^{-ifl}) \\ &= \frac{4}{pf} \sin pl \sin fl \end{aligned}$$

(5)

Assuming constant gain over the illuminated area and negligible variation for S in the integrand, we have then some trivial integration over the variables, S and φ . The final result for this term in view of (4), (5), and (2) is

$$\begin{aligned} & K_1 \frac{\beta \Gamma S_0 A_0 e^{-K}}{2 \sin \alpha} \left[\frac{4}{pf} \sin pl \sin fl + \sum_{n=1}^{\infty} \frac{K^n}{n!} 2\pi n a^2 [n^2 + a^2(p^2 + f^2)]^{-3/2} \right] \\ &= K \frac{\alpha A_0 e^{-K}}{\sin \alpha} \left[\frac{2}{\pi \lambda^2 pf} \sin pl \sin fl \right. \\ & \quad \left. - \sum_{n=1}^{\infty} \frac{K^n}{(n-1)!} \left(\frac{a}{\lambda}\right)^2 [n^2 + a^2(p^2 + f^2)]^{-3/2} \right] \end{aligned} \quad (6)$$

where $K = 2\eta H_0^2 \frac{c\tau S\pi}{R}$

and note that $\beta = 4\alpha$

(II) Consider the integral

$$-K_1 \iiint \frac{i 2k B_0 \sigma^2 \sin \theta'}{a} \exp\left[-\frac{\xi}{a} - i\gamma \xi - K(1 - e^{-\xi/a})\right] \cdot S_0 \csc \alpha \xi d\xi d\theta' d\tau \quad (7)$$

where $B_0 = (\cos \theta + \cos \alpha) \sin \theta \cos \theta \sin \phi$

$$= -K_1 i 2k B_0 \sigma^2 \frac{S_0 \csc \alpha e^{-K}}{a} \sum_{n=0}^{\infty} \frac{K^n}{n!} \iiint \sin \theta' \exp\left[-(n+1)\frac{\xi}{a} - i\gamma \xi\right] \xi d\xi d\theta' d\tau \quad (8)$$

The integral with respect to ξ and θ' is

$$\iint \sin \theta' \exp\left[-\left(\frac{n+1}{a} + i\gamma\right)\xi\right] \xi d\xi d\theta' = \int_0^{2\pi} \frac{\sin \theta' d\theta'}{\left(\frac{n+1}{a} - \gamma\right)^2} \quad (9)$$

Transform onto the unit circle we get

$$\oint \left(\frac{z - z^{-1}}{2i}\right) \frac{dz}{iz \left[\frac{n+1}{a} + \frac{\rho}{2}(z - z^{-1}) - i\frac{\gamma}{2}(z + z^{-1})\right]^2}$$

$$= -\frac{2}{(p-ig)^2} \oint \frac{(z^2-1) dz}{(z-z_1)^2 (z-z_2)^2}$$

where

$$\left. \begin{array}{l} z_1(+) \\ z_2(-) \end{array} \right\} \equiv \frac{-\frac{n+1}{a} \pm \sqrt{\left(\frac{n+1}{a}\right)^2 + p^2 + q^2}}{p-ig}$$

Residue at $z = z_1$ is

$$\begin{aligned} & -\frac{2}{(p-ig)^2} \frac{d}{dz} \left[\frac{z^2-1}{(z-z_2)^2} \right]_{z=z_1} \\ &= \frac{4}{(p-ig)^2} \frac{z z_2 - 1}{(z-z_2)^3} \Big|_{z=z_1} \\ &= \frac{-4}{(p-ig)^2} \frac{[(p-ig)^2 + (p^2+q^2)] / (p-ig)^2}{8 \left[\left(\frac{n+1}{a}\right)^2 + p^2 + q^2 \right]^{3/2} / (p-ig)^3} \\ &= -\frac{1}{2(p-ig)} \frac{[(p-ig)^2 + p^2 + q^2]}{\left[\left(\frac{n+1}{a}\right)^2 + p^2 + q^2 \right]^{3/2}} \\ &= \frac{-p}{\left[\left(\frac{n+1}{a}\right)^2 + p^2 + q^2 \right]^{3/2}} \end{aligned} \tag{10}$$

Hence (9) becomes
$$\frac{-2\pi i p}{\left[\left(\frac{n+1}{a}\right)^2 + p^2 + q^2 \right]^{3/2}} \tag{11}$$

The final result of (8) then becomes in view of (11)

$$\frac{K_1 \beta C T S_0 B_0}{2 \sin \alpha} \frac{4\pi k p \sigma^2}{a} e^{-k} \sum_{n=0}^{\infty} \frac{K^n}{n!} \left[\left(\frac{n+1}{a}\right)^2 + p^2 + q^2 \right]^{-3/2}$$

$$= -R \frac{e^{\kappa} 8\pi^2 \alpha B_0 (\sin \alpha - \sin \theta \sin \phi)}{\sin \alpha} \left(\frac{a\sigma}{\lambda^2}\right)^2 \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} [(n+1)^2 + a^2(p^2 + q^2)]^{-3/2} \quad (12)$$

where $B_0 = (\cos \theta + \cos \alpha) \sin \theta \cos \theta \sin \phi$

(III) Consider the integral

$$-K_1 C_0 \sigma^2 / a^2 \iiint \sin^2 \theta' \exp[-i\gamma \xi - \kappa(1 - e^{-\xi/a}) - \xi/a] \\ S_0 \cos \alpha \xi d\xi d\theta' ds d\gamma \quad (13)$$

where $C_0 = \sin^2 \theta$

$$= -K_1 \frac{S_0 C_0 \sigma^2}{a^2 \sin \alpha} e^{\kappa} \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} \iiint e^{-\frac{n+1}{a}\xi - i\gamma \xi} \sin^2 \theta' \xi d\xi d\theta' ds d\gamma \\ = -K_1 \frac{C_0 \sigma^2 S_0}{a^2 \sin \alpha} e^{-\kappa} \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} \iiint_0^{2\pi} \frac{\sin^2 \theta' d\theta' d\gamma ds}{\left(\frac{n+1}{a} + i\gamma\right)^2} \quad (14)$$

In (14) the value of the integral,

$$\int_0^{2\pi} \frac{\sin^2 \theta' d\theta'}{\left(\frac{n+1}{a} + i\gamma \sin \theta' - i\gamma \cos \theta'\right)^2} \\ = \oint \frac{iZ (Z^2 - 2 + Z^{-1}) dZ}{4 (Z - Z_1)^2 (Z - Z_2)^2 \frac{1}{4} (p - iq)^2} \\ = \oint \frac{i (Z^4 - 2Z^2 + 1) dZ}{(p - iq)^2 Z (Z - Z_1)^2 (Z - Z_2)^2}$$

where z_1, z_2 are as defined in (II)

Residue at $z = 0$ is

$$\begin{aligned}
 &= \frac{i}{(p-iq)^2 z_1^2 z_2^2} = \frac{i(p-iq)}{(p^2+q^2)^2 (p-iq)^2} \\
 &= \frac{i}{(p+iq)^2}
 \end{aligned} \tag{16}$$

$(p-iq)^2$ times residue at $z = z_1$ gives

$$\begin{aligned}
 &\frac{d}{dz} \left[\frac{i(z^4 - 2z^2 + 1)}{z(z-z_2)^2} \right]_{z=z_1} \\
 &= i \left[\frac{z(z^2 - 2z_2z + z_2^2)(4z^3 - 4z) - (z^4 - 2z^2 + 1)[z^2 - 2z_2z + z_2^2 + 2z(z-z_2)]}{z^2(z-z_2)^4} \right]_{z=z_1} \\
 &= i \left[\frac{z(z^2 - 2z_2z + z_2^2)4(z^2 - z) - (z^4 - 2z^2 + 1)[(z-z_2)(3z - z_2)]}{z^2(z-z_2)^4} \right]_{z=z_1} \\
 &= i \left[\frac{(z^2 - 1)[4z^2(z-z_2) - (z^2 - 1)(3z - z_2)]}{z^2(z-z_2)^3} \right]_{z=z_1} \\
 &= i \left[\frac{(z^2 - 1)[z^2(z - 3z_2) + 3(z - z_2)]}{z^2(z-z_2)^3} \right]_{z=z_1} \\
 &= i \left[\frac{z^2(z - 3z_2) + (3z - z_2)}{(z-z_2)^3} - \frac{z - 3z_2}{(z-z_2)^3} - \frac{3z - z_2}{z^2(z-z_2)^3} \right]_{z=z_1} \\
 &= i \left[\frac{z_1^2(z_1 - 3z_2)}{(z_1 - z_2)^3} + \frac{2(z_1 + z_2)}{(z_1 - z_2)^3} - \frac{3z_1 - z_2}{z_1^2(z_1 - z_2)^3} \right]
 \end{aligned}$$

$$\begin{aligned}
&= \frac{i(p-ig)^3}{8 \left[\left(\frac{n+1}{a} \right)^2 + p^2 + q^2 \right]^{3/2}} \left\{ \frac{\left(R' - \frac{n+1}{a} \right)^2}{(p-ig)^2} \left[\frac{2 \frac{n+1}{a} + 4R'}{p-ig} \right] \right. \\
&\quad \left. - \frac{4 \frac{n+1}{a}}{p-ig} - \left[\frac{4R' - 2 \frac{n+1}{a}}{p-ig} \right] \left[\frac{p-ig}{-\frac{n+1}{a} + R'} \right] \right\} \quad (17)
\end{aligned}$$

where $R' = \left[\left(\frac{n+1}{a} \right)^2 + p^2 + q^2 \right]^{1/2}$

Thus (15) is given by $2\pi i$ times the sum of the residues at $z=0$ and $z=z_1$.

Further integration with respect to S and Υ leads to the following final result for (13),

$$\begin{aligned}
&\mathbb{K} \frac{\alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda} \right)^2 e^{-\kappa} \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} \left[\frac{1}{a^2 (p+ig)^2} \right. \\
&\quad - \frac{n+1}{2 \left[(n+1)^2 + p^2 + q^2 \right]^{3/2}} - \frac{1}{8 \left[(n+1)^2 + a^2 (p^2 + q^2) \right]^{3/2}} \\
&\quad \cdot \left\{ \left(\frac{R' - \frac{n+1}{a}}{p-ig} \right)^2 \left[2(n+1) + 4 \sqrt{(n+1)^2 + a^2 (p^2 + q^2)} \right] \right. \quad (18) \\
&\quad \left. \left. + \left(\frac{p-ig}{-\frac{n+1}{a} + R'} \right)^2 \left[2(n+1) - 4 \sqrt{(n+1)^2 + a^2 (p^2 + q^2)} \right] \right\} \right]
\end{aligned}$$

(IV) Consider now the integral of the form,

$$K_1 C_0 \frac{\sigma^2}{a} \iiint \frac{\cos^2 \theta'}{\xi} \exp \left[-i\gamma \xi - K \left(1 - e^{-\xi/a} \right) - \frac{\xi}{a} \right]$$

$$S_0 \cos \alpha \xi d\xi d\theta' ds d\tau$$

$$= \frac{K_1 C_0 \sigma^2 S_0 e^{-K}}{a \sin \alpha} \sum_{n=0}^{\infty} \frac{K^n}{n!} \iiint \cos^2 \theta' \exp \left[-i\gamma \xi - \frac{(n+1)\xi}{a} \right]$$

$$\cdot d\xi d\theta' ds d\tau$$

$$= \frac{K_1 C_0 \sigma^2 S_0 e^{-K}}{a \sin \alpha} \sum_{n=0}^{\infty} \frac{K^n}{n!} \iiint \frac{\cos^2 \theta' d\theta'}{\left[\frac{n+1}{a} + ip \sin \theta' - iq \cos \theta' \right]} ds d\tau$$

(19)

Let us consider first the integration with respect to θ' i.e.

$$\begin{aligned} & \int_0^{2\pi} \frac{\cos^2 \theta' d\theta'}{\left[\frac{n+1}{a} + ip \sin \theta' - iq \cos \theta' \right]} \\ &= \oint \frac{\left(\frac{z+z^{-1}}{2} \right)^2 \frac{dz}{iz}}{\frac{n+1}{a} + ip \left(\frac{z-z^{-1}}{2i} \right) - iq \frac{z+z^{-1}}{2}} \\ &= \oint \frac{-i(z+z^{-1})^2/4 dz}{\frac{1}{2}(p-iq)z^2 + \frac{n+1}{a}z - \frac{1}{2}(p+iq)} \\ &= \oint \frac{-i(z^4 + 2z^2 + 1) dz}{2(p-iq)(z-z_1)(z-z_2)z^2} \end{aligned}$$

(20)

Residue at $Z = 0$ is

$$\frac{-i}{2(p-iq)} \left[\frac{(Z-Z_1)(Z-Z_2)(Z^2+1) + Z - (Z^2+1)^2 [(Z-Z_1) + Z - Z_2]}{(Z-Z_1)^2 (Z-Z_2)^2} \right]_{Z=0}$$

$$= -\frac{i}{2(p-iq)} \frac{Z_1 + Z_2}{Z_1^2 Z_2^2} \quad (21)$$

Note

$$Z_1 + Z_2 = -\frac{2(n+1)}{a(p-iq)}$$

$$Z_1 Z_2 = \frac{-(p^2 + q^2)}{(p-iq)^2}$$

$$Z_1 - Z_2 = \frac{2 \left[\left(\frac{n+1}{a} \right)^2 + p^2 + q^2 \right]^{1/2}}{p-iq}$$

Hence (21) becomes

$$\frac{i}{2(p-iq)} \frac{2(n+1) (p-iq)^4}{a(p-iq) (p^2 + q^2)^2}$$

$$= \frac{i(n+1)}{a(p+iq)^2} \quad (22)$$

Residue at $Z = Z_1$ is

$$\frac{-i(Z_1^4 + 2Z_1^2 + 1)}{2(p-iq) Z_1^2 (Z_1 - Z_2)}$$

$$= -\frac{i}{2(p-iq)} \left[\frac{Z_1^2}{Z_1 - Z_2} + \frac{2}{Z_1 - Z_2} + \frac{1}{Z_1^2 (Z_1 - Z_2)} \right]$$

$$\begin{aligned}
&= \frac{-i}{2(p-ig)} \left[\left(\frac{R' - \frac{n+1}{a}}{p-ig} \right)^2 \frac{(p-ig)}{2 \left[\left(\frac{n+1}{a} \right)^2 + p^2 + q^2 \right]^{1/2}} - \frac{p-ig}{\left[\left(\frac{n+1}{a} \right)^2 + p^2 + q^2 \right]^{1/2}} \right. \\
&\quad \left. + \left(\frac{p-ig}{-\frac{n+1}{a} + R'} \right)^2 \frac{p-ig}{2 \left[\left(\frac{n+1}{a} \right)^2 + p^2 + q^2 \right]^{1/2}} \right] \\
&= \frac{-ia}{2 \left[(n+1)^2 + a^2(p^2+q^2) \right]^{1/2}} - \frac{i}{4 \left[(n+1)^2 + a^2(p^2+q^2) \right]^{1/2}} \\
&\quad \cdot \left\{ \frac{[-(n+1) + \sqrt{(n+1)^2 + a^2(p^2+q^2)}]^2}{a(p-ig)^2} + \frac{a^3(p-ig)^2}{[-(n+1) + \sqrt{(n+1)^2 + a^2(p^2+q^2)}]^2} \right\} \quad (23)
\end{aligned}$$

Final result for (19) after further integration with respect to S and δ is

$$\begin{aligned}
&\frac{K_1 C_0 \sigma^2 S_0 c T \beta}{2a \sin \alpha} e^{-k} \sum_{n=0}^{\infty} \frac{K^n}{n!} 2\pi \left[-\frac{(n+1)}{a(p+ig)^2} + \frac{a}{2R'} \right. \\
&\quad \left. + \frac{1}{4R'} \left\{ \frac{[R' - (n+1)]^2}{a(p-ig)^2} - \frac{a^3(p-ig)^2}{[R' - (n+1)]^2} \right\} \right] \\
&= K \frac{\alpha \sin^2 \theta}{\sin \alpha} \left(\frac{\sigma}{\lambda} \right)^2 e^{-k} \sum_{n=0}^{\infty} \frac{K^n}{n!} \left\{ -\frac{n+1}{a^2(p+ig)^2} + \frac{1}{2R'} \right. \\
&\quad \left. + \frac{1}{4R'} \left(\frac{[R' - (n+1)]^2}{a^2(p-ig)^2} + \frac{a^2(p-ig)^2}{[R' - (n+1)]^2} \right) \right\} \quad (24)
\end{aligned}$$

where $R' = \left[(n+1)^2 + a^2(p^2+q^2) \right]^{1/2}$

APPENDIX 5

THE LIMITING VALUE OF (III-24) AS

$$\phi \rightarrow \pi/2, \quad \alpha = 0 \quad \text{AND} \quad p, q \rightarrow 0.$$

Consider terms of the form

$$\begin{aligned} [R' - (n+1)]^2 &= [-(n+1) + (n+1) \sqrt{1 + \frac{a^2(p^2+q^2)}{(n+1)^2}}]^2 \\ &\rightarrow \left[\frac{a^2(p^2+q^2)}{2(n+1)} \right]^2 \end{aligned}$$

$$\begin{aligned} \therefore \left[\frac{a(p-iq)}{R' - (n+1)} \right]^2 [n+1 - 2R'] & \\ = \frac{a^2(p-iq)^2}{a^4(p^2+q^2)^2} 4(n+1)^2 [-(n+1)] & \\ = \frac{-4(n+1)^3}{a^2(p+iq)^2} & \end{aligned}$$

$$\therefore \frac{1}{4R'^3} \left[\frac{a(p-iq)}{R' - (n+1)} \right]^2 [n+1 - 2R'] = \frac{-1}{a^2(p+iq)^2} \quad (1)$$

$$\therefore \frac{1}{4R'} \left[\frac{a(p-iq)}{R' - (n+1)} \right]^2 = \frac{n+1}{a^2(p+iq)^2} \quad (2)$$

From (1) and (2), (III-24) becomes in the limit

$$K \frac{\alpha \cos^2 \alpha}{\sin \alpha} e^{-k} \left[\frac{2l^2}{\pi \lambda^2} + \left(\frac{a}{\lambda} \right)^2 \sum_{n=1}^{\infty} \frac{K^n}{n^2 n!} \right]$$

$$+ K \alpha \sin \alpha \left(\frac{\sigma}{\lambda}\right)^2 e^{-K} \sum_{n=0}^{\infty} \frac{K^n}{n!} \left\{ -\frac{1}{2(n+1)^2} + \frac{1}{2(n+1)} \right\}$$

$$+ K 16\pi^2 \alpha \sin \alpha \cos^2 \alpha \left(\frac{\sigma}{\lambda}\right)^4 e^{-K} \sum_{n=0}^{\infty} \frac{K^n}{n!} \left\{ \frac{-1}{2(n+2)^2} \right\}$$

APPENDIX 6

EVALUATION OF (III-23) FOR THE CASE OF LARGE K

(I) Consider the integral

$$K_1 \iiint A_0 \exp[-i\gamma \xi - K(1 - e^{-\xi/a})] S_0 \cos \alpha \xi d\xi d\theta' d\sigma d\tau$$

$$\approx K_1 \iiint A_0 \exp[-i\gamma \xi - K\xi/a] S_0 \cos \alpha \xi d\xi d\theta' d\sigma d\tau \quad (1)$$

where

$$\gamma = -p \sin \theta' + q \cos \theta'$$

$$K = k^2 \sigma^2 (\cos \theta + \cos \alpha)^2$$

The two integrals with respect to ξ and θ' is

$$\iint \exp[-i\gamma \xi - K\xi/a] \xi d\xi d\theta'$$

$$= \int_0^{2\pi} \frac{d\theta'}{\left(\frac{K}{a} + ip \sin \theta' - iq \cos \theta'\right)^2} \quad (2)$$

Let

$$d\theta' = \frac{dz}{iz}, \quad \cos \theta' = (z + z^{-1})/2, \quad \sin \theta' = (z - z^{-1})/2i$$

(2) becomes

$$\oint \frac{-iz dz}{\left[\left(\frac{p}{2} - i\frac{q}{2}\right)z^2 + \frac{K}{a}z - \left(\frac{p}{2} + i\frac{q}{2}\right)\right]^2}$$

$$= \oint \frac{-4iz dz}{(p - iq)^2 (z - z_1)^2 (z - z_2)^2}$$

where

$$\left. \begin{array}{l} z_1(+), \\ z_2(-) \end{array} \right\} = \frac{-\frac{K}{a} \pm \sqrt{\left(\frac{K}{a}\right)^2 + a^2(\rho^2 + \xi^2)}}{(\rho - i\xi)}$$

Residue at $z = z_1$ is

$$\begin{aligned} & \frac{d}{dz} \left[\frac{-iz}{(z - z_2)^2} \right]_{z=z_1} \frac{4}{(\rho - i\xi)^2} \\ &= \frac{i(z_1 + z_2)}{(z_1 - z_2)^3} \frac{4}{(\rho - i\xi)^2} \\ &= -i \frac{K}{a} \frac{1}{\left[\left(\frac{K}{a}\right)^2 + \rho^2 + \xi^2 \right]^{3/2}} \end{aligned} \quad (3)$$

The integral in (2) is then given by

$$\frac{2\pi K a^2}{[K^2 + a^2(\rho^2 + \xi^2)]^{3/2}}$$

and the final result for (1) after further integration with respect to S and γ is

$$\begin{aligned} & K_1 A_0 \beta \frac{cT}{2} \frac{2S_0 \pi}{\sin \alpha} \frac{K a^2}{[K^2 + a^2(\rho^2 + \xi^2)]^{3/2}} \\ &= K \frac{\alpha(1 - \sin^2 \theta \sin^2 \phi)}{\sin \alpha} \left(\frac{a}{\lambda}\right)^2 \frac{K}{[K^2 + a^2(\rho^2 + \xi^2)]^{3/2}} \end{aligned} \quad (4)$$

where
$$K = 4cT S_0 \pi k_1 \lambda^2$$

$$= 207 H_0^2 cT S_0 \pi / R_0^2$$

Note the way this integral is evaluated is the same as the corresponding one in Appendix 4 with n replaced by K and leave out the factor

$$e^{-K} \sum_{n=0}^{\infty} \frac{K^n}{n!} \quad . \quad \text{Thus, we need not do other integrals,}$$

but only have to use the results of Appendix 4 accordingly.

A particular approximate expression for backscattering for this case when $K \gg 1$ is as follows

$$\begin{aligned} \bar{P} = & K \frac{1}{(4\pi)^2} \frac{\alpha (1 + \sin^2 \alpha)}{\sin \alpha \cos^4 \alpha} A (1 + A')^{-3/2} \\ & + K \frac{1}{(4\pi)^2} \frac{\alpha \sin \alpha}{\alpha \omega^2 \alpha} \left\{ \frac{1}{(1 + A')^{1/2}} \left(\frac{1}{2} + \frac{[-1 + (1 + A')^{1/2}]^2}{4A'} \right. \right. \\ & \left. \left. + \frac{A'}{4[-1 + (1 + A')^{1/2}]^2} \right) \right\} \\ & + K \frac{1}{(4\pi)^4} \left(\frac{\lambda}{\sigma} \right)^2 \left\{ \frac{\alpha \sin \alpha}{\cos^4 \alpha} \frac{1}{(1 + A')^{3/2}} \left[-\frac{1}{2} \right. \right. \\ & + \frac{1}{4} \left(\frac{[-1 + (1 + A')^{1/2}]^2}{A'} [1 + 2(1 + A')^{1/2}] \right. \\ & \left. \left. + \frac{A'}{[-1 + (1 + A')^{1/2}]^2} [1 - 2(1 + A')^{1/2}] \right) \right] \right\} \end{aligned}$$

$$\begin{aligned}
& + \left[\frac{1}{(4\pi)^2} \frac{\alpha \sin \alpha}{\cos^2 \alpha} \frac{1}{(1+A')^{3/2}} \left[-\frac{1}{2} \right. \right. \\
& + \frac{1}{4} \left(\frac{[-1 + (1+A')^{1/2}]^2}{A'} [1 + 2(1+A')^{1/2}] \right. \\
& \left. \left. + \frac{A'}{[-1 + (1+A')^{1/2}]^2} [1 - 2(1+A')^{1/2}] \right) \right] \quad (5)
\end{aligned}$$

where $A = \left(\frac{a\lambda}{4\pi r^2} \right)^2$

$$A' = A \frac{\sin^2 \alpha}{\cos^4 \alpha}$$

APPENDIX 7

MOTIVATION FOR THE PARTICULAR FORM OF THE
SURFACE-HEIGHT CORRELATION FUNCTION

The result of Daniels [1961] shows that the backscattered power and the signal correlation function is related through Hankel Transformation, i. e.

$$P(\alpha) = \int_0^{\infty} \cos^2 \alpha P_E(\xi) \xi J_0(2k\xi\alpha) d\xi \quad (1)$$

where $P(\alpha)$ is the average return power

$P_E(\xi)$ the signal correlation function, = $\exp \{-K [1 - r(\xi)]\}$

$$K = 4k^2\sigma^2 \cos^2 \alpha$$

σ = standard deviation of the surface

$$k = 2\pi/\lambda$$

λ = wavelength

$r(\xi)$ = surface-height correlation coefficient

J_0 = zero order Bessel function

Thus, the inverse transform gives

$$\begin{aligned} P_E(\xi) &= \int_0^{\infty} J_0(2k\alpha\xi) \left[\frac{P(\alpha)}{\cos^2 \alpha} \right] (2k\alpha) d(2k\alpha) \\ &= \int_0^{\infty} \frac{4k^2}{\cos^2 \alpha} J_0(2k\alpha\xi) P(\alpha) \alpha d\alpha \end{aligned}$$

$$-K [1 - r(\xi)] = \ln \left[\int_0^{\infty} \frac{4k^2}{\cos^2 \alpha} J_0(2k\alpha\xi) P(\alpha) \alpha d\alpha \right]$$

$$r(\xi) = 1 + \frac{1}{K} \ln \left[\int_0^{\infty} \frac{4k^2}{\cos^2 \alpha} J_0(2k\alpha\xi) P(\alpha) \alpha d\alpha \right] \quad (2)$$

It is, thus, seen from (2) that the surface-height correlation coefficient should have the form as given by (2).

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